6.1 Square Roots

The **square** of a number is the product of that number and itself. When a number is squared, it is raised to a second power. Consider the expression 8^2. The **exponent**, 2, with **base**, 8, shows that the number is squared, or appears as a factor twice.

For example: $8^2 = 8 \times 8 = 64$

Finding a number that has been squared is called finding the square root of the number. The symbol $\sqrt{}$ is used to indicate a square root.

For example: $\sqrt{64} = \sqrt{8 \times 8} = 8$

Squaring a number, and finding the square root of a number, can be thought of as inverses of each other. When squaring a number, a new product is produced, and taking the square root of the new product produces the original number.

For example: Squaring 5 results in 25, because $5^2 = 5 \times 5 = 25$.
The square root of 25 is 5, because $\sqrt{25} = \sqrt{5 \times 5} = 5$

Perfect Square Numbers

Whole numbers are the numbers: $0, 1, 2, 3, \ldots$. A perfect square number is a number whose square root is a whole number.

For example: $\sqrt{0} = 0$, since $0^2 = 0 \times 0 = 0$

$\sqrt{1} = 1$, since $1^2 = 1 \times 1 = 1$

$\sqrt{4} = 2$, since $2^2 = 2 \times 2 = 4$

$\sqrt{9} = 3$, since $3^3 = 3 \times 3 = 9$

$\sqrt{16} = 4$, since $4^2 = 4 \times 4 = 16$

$\vdots$

$\sqrt{a^2} = a,\ a \geq 0$, since $a^2 = a \times a$

A partial list of perfect squares: 0, 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, ...

Square roots of numbers that are not perfect squares are **irrational numbers**. In decimal form, these numbers continue forever in a non repeating pattern.

For example: $\sqrt{12} = 3.4641016151\ldots$

Squaring and Square Root Properties

Squaring and square root properties can be used to solve equations containing squares and square roots. An important concept to understand for solving these equations is that squaring a square root produces the original value, or in other words, one cancels the other out.

For example: $(\sqrt{16})^2 = (\sqrt{4\times4})^2 = (4)^2 = 16$.

Example 1 Solve for x, $x \geq 0$

a) $\sqrt{x} = 16$

b) $\sqrt{x^2} = 9$

c) $\sqrt{x} = 2^2$

d) $\sqrt{x^2} = \sqrt{25}$

► *Solution*:

a)
$$\sqrt{x} = 16$$
$$(\sqrt{x})^2 = (16)^2$$
$$x = 256$$

b)
$$\sqrt{x^2} = 9$$
$$(\sqrt{x^2})^2 = 9^2$$
$$x^2 = 9^2$$
$$x = 9$$

c)
$$\sqrt{x} = 2^2$$
$$\sqrt{x} = 4$$
$$(\sqrt{x})^2 = 4^2$$
$$x = 16$$

d)
$$\sqrt{x^2} = \sqrt{25}$$
$$(\sqrt{x^2})^2 = (\sqrt{25})^2$$
$$x^2 = 25$$
$$x = 5$$

It is not too difficult to find the square root of a small perfect square; however, finding the square root of a large perfect square can be much more challenging.

One way of simplifying a square root is using the properties: $\sqrt{a\times b} = \sqrt{a}\times\sqrt{b}$ and $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.

For example: $\sqrt{36} = \sqrt{4\times9} = \sqrt{4}\times\sqrt{9} = 2\times3 = 6$.

$$\sqrt{\frac{9}{25}} = \frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}$$

Example 2 Find the square root.

a) $\sqrt{1296}$

b) $\sqrt{1764}$

c) $\sqrt{\frac{98}{242}}$

► *Solution*:

a)
$$\sqrt{1296} = \sqrt{4\times324}$$
$$= \sqrt{4\times4\times81}$$
$$= \sqrt{4}\times\sqrt{4}\times\sqrt{81}$$
$$= 2\times2\times9$$
$$= 36$$

b)
$$\sqrt{1764} = \sqrt{4\times441}$$
$$= \sqrt{4\times9\times49}$$
$$= \sqrt{4}\times\sqrt{9}\times\sqrt{49}$$
$$= 2\times3\times7$$
$$= 42$$

c)
$$\sqrt{\frac{98}{242}} = \frac{\sqrt{98}}{\sqrt{242}}$$
$$= \frac{\sqrt{2\times49}}{\sqrt{2\times121}}$$
$$= \frac{\cancel{\sqrt{2}}\times\sqrt{49}}{\cancel{\sqrt{2}}\times\sqrt{121}}$$
$$= \frac{7}{11}$$