1 Measurement

BUILDING ON

- working with metric units of measure
- determining the surface areas and volumes of right prisms and cylinders

BIG IDEAS

- You can use proportional reasoning to convert measurements.
- The volume of a right pyramid or cone is related to the volume of the enclosing right prism or cylinder.
- The surface area of a right pyramid or cone is the sum of the areas of the faces and the curved surfaces.
- The surface area of a sphere is related to the curved surface area of the enclosing cylinder.

NEW VOCABULARY

imperial units
unit analysis
SI system of measures
apex
right pyramid and right cone
slant height
lateral area
sphere

STARSHIP ENTERPRISE

A replica was constructed in Vulcan, Alberta, in June 1995. The base is part of a pyramid and it supports a 31-ft. long spacecraft. Visitors are welcomed in 3 languages: English, Vulcan, and Klingon.

1.1 Imperial Measures of Length

LESSON FOCUS

Develop personal referents to estimate imperial measures of length.

Make Connections

In 1976, Canada adopted SI units to measure length. However, construction and manufacturing industries continue to use **imperial units**. Many Canadians use imperial units to measure their height.

What is your height?

The **SI system of measures** is an abbreviation for *Le Système International d'Unités*. Since 1960, this form of the metric system has been adopted by many countries, including Canada.

Some **imperial units** of measure are the inch, the foot, the yard, and the mile.

Look around the classroom.
Which object has a length of about one foot?
Which object has a length of about one inch?
Which object has a length of about one yard?

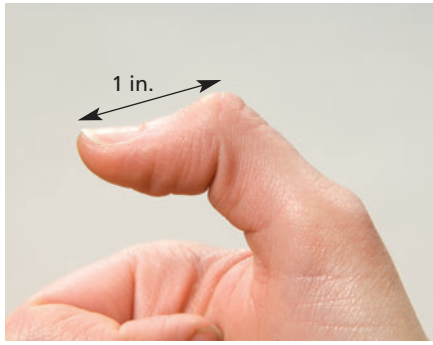
Construct Understanding

Many units in the imperial system are based on measurements of the human body.

TRY THIS

Work with a partner.

You will need a ruler and measuring tape with imperial units, string, and scissors.



A. Use the length of your thumb from its tip to the first joint as a measuring instrument.
Use your thumb to estimate then record the length of your pencil and the dimensions of your textbook, in thumb units.
Use a ruler or measuring tape to measure each length.
For which imperial unit could you use your thumb length as a referent?

B. Use your foot length to estimate then record the length and width of a bookcase, in foot units.
Use a ruler or measuring tape to measure each length.
For which imperial unit could you use your foot length as a referent?

C. Hold a piece of string from your nose to the longest finger of an outstretched arm. Have your partner cut the string to this length.
Use this string to estimate then record the length and width of the classroom, in arm spans. Use a measuring tape to measure each length.
For which imperial unit could you use your arm span as a referent?

D. Determine approximately:

- how many thumb lengths equal one foot length
- how many foot lengths equal one arm span
- how many thumb lengths equal one arm span

E. Compare the measures in your personal units with those of your partner. Why is it necessary to have standard units of length?

F. What other referents could be used for each length?

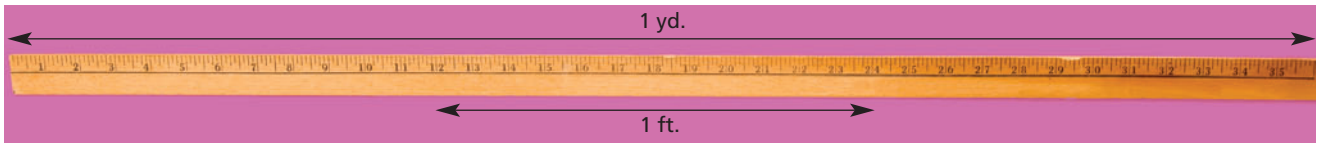
- one inch
- one foot
- one yard

People have been measuring for thousands of years. Early trades people created their own measuring devices and units. For example, people measured the distance between two places by the number of days it took to travel from one place to the other. Over time, these units were standardized as imperial units, and relationships between the units were determined.

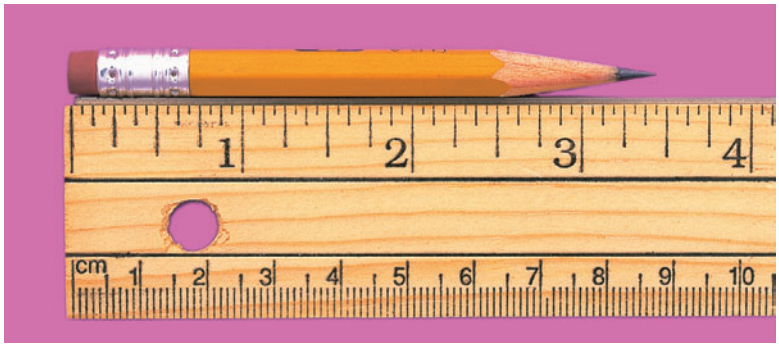
The imperial unit for measuring long distances is the mile. The length of one mile was first established as the distance a Roman soldier could walk in 1000 paces. One pace is 2 steps.

On maps and scale diagrams, you may see symbols for imperial units instead of abbreviations. One inch is 1″ and one foot is 1′.

Imperial Unit	Abbreviation	Referent	Relationship between Units
Inch	in.	Thumb length	
Foot	ft.	Foot length	1 ft. = 12 in.
Yard	yd.	Arm span	1 yd. = 3 ft. 1 yd. = 36 in.
Mile	mi.	Distance walked in 20 min	1 mi. = 1760 yd. 1 mi. = 5280 ft.



Many rulers marked with imperial units show one inch divided into eighths, tenths, or sixteenths. To measure the length of an object, you must first determine the smallest indicated unit by counting the number of divisions between two adjacent inch marks. The ruler below has 16 divisions between two adjacent inch marks, so the smallest indicated unit is $\frac{1}{16}$ of an inch, which is written as $\frac{1}{16}$ in.



The pencil point is closest to the 7th division mark between 3 in. and 4 in., so its length is $3\frac{7}{16}$ in.

A fraction of an imperial measure of length is usually written in fraction form, not decimal form.

Example 1 Converting between Imperial Units

a) Convert 5 yd. to:

i) feet **ii)** inches

b) Convert 51 in. to:

i) feet and inches **ii)** yards, feet, and inches

SOLUTION

a) i) Since 1 yd. = 3 ft., to convert yards to feet multiply by 3.

5 yd. = 5(3 ft.)

5 yd. = 15 ft.

ii) Since 5 yd. = 15 ft. and 1 ft. = 12 in., to convert feet to inches multiply by 12.

5 yd. = 15(12 in.)

5 yd. = 180 in.

b) i) Since 12 in. = 1 ft., to convert inches to feet, divide by 12.

$51 \text{ in.} = \frac{51}{12} \text{ ft.}$ Write this improper fraction as a mixed number.

$51 \text{ in.} = 4\frac{3}{12} \text{ ft.}$

51 in. = 4 ft. 3 in.

ii) 51 in. = 4 ft. 3 in.

Since 3 ft. = 1 yd.

4 ft. = 1 yd. 1 ft.

and 51 in. = 1 yd. 1 ft. 3 in.

CHECK YOUR UNDERSTANDING

1. a) Convert 7 yd. to:

i) feet **ii)** inches

b) Convert 62 in. to:

i) feet and inches

ii) yards, feet, and inches

[Answers: a) i) 21 ft. ii) 252 in.
b) i) 5 ft. 2 in. ii) 1 yd. 2 ft. 2 in.]

When you convert a measurement from a larger unit to a smaller unit, do you expect the number of units to increase or decrease? Why?

There are 3 ft. in 1 yd. so, in *Example 1*, we multiply by 3 to convert a measurement in yards to a measurement in feet; this is one type of *proportional reasoning*.

Similarly, we use proportional reasoning when we divide by 12 to convert a measurement in inches to a measurement in feet.

We also use proportional reasoning when we write then solve a proportion, as illustrated on the next page.

Example 2 Solving a Problem Involving Converting between Units

Anne is framing a picture.
The perimeter of the framed picture will be 136 in.

a) What will be the perimeter of the framed picture in feet and inches?

b) The framing material is sold by the foot. It costs \$1.89/ft. What will be the cost of material before taxes?

SOLUTIONS

a) Method 1

To convert inches to feet, divide by 12.

$$136 \text{ in.} = \frac{136}{12}\text{ft.}$$
$$136 \text{ in.} = 11\frac{4}{12}\text{ ft.}$$

So, 136 in. = 11 ft. 4 in.

The perimeter of the framed picture will be 11 ft. 4 in.

Method 2

Use a proportion. Let x represent the length in feet.

The ratio of x feet to 136 in. is equal to the ratio of 1 ft. to 12 in. Write a proportion.

$$\frac{x}{136} = \frac{1}{12}$$ Multiply each side by 136.

$$136\left(\frac{x}{136}\right) = 136\left(\frac{1}{12}\right)$$
$$x = \frac{136}{12}$$
$$x = 11\frac{4}{12}$$

$11\frac{4}{12}$ ft. is 11 ft. 4 in.

The perimeter of the framed picture will be 11 ft. 4 in.

b) The perimeter of the framed picture is greater than 11 ft., so Anne must buy 12 ft. of framing material.

The cost, C, is:
$C = 12(\$1.89)$
$C = \$22.68$

Before taxes, the material will cost \$22.68.

CHECK YOUR UNDERSTANDING

2. Ben buys baseboard for a bedroom. The perimeter of the bedroom, excluding closets and doorway, is 37 ft.

a) What length of baseboard is needed, in yards and feet?

b) The baseboard material is sold by the yard. It costs \$5.99/yd. What is the cost of material before taxes?

[Answers: a) 12 yd. 1 ft. b) \$77.87]

How could you use mental math and estimation to check that your answer is reasonable?

We can use **unit analysis** to verify a conversion between units.

Unit analysis is one method of verifying that the units in a conversion are correct.

For *Example 2*, we can write the relationship between feet and inches as a fraction in two ways: $\frac{1 \text{ ft.}}{12 \text{ in.}}$ and $\frac{12 \text{ in.}}{1 \text{ ft.}}$

These fractions are *conversion factors*. Each fraction is equal to 1, so we can multiply any number by the fraction and not alter its value.

Since we are converting 136 in. to feet, we use the conversion factor with feet in the numerator. We write:

$$136 \text{ in.} \times \frac{1 \text{ ft.}}{12 \text{ in.}} = \frac{136 \text{ in.}}{1} \times \frac{1 \text{ ft.}}{12 \text{ in.}}$$ Eliminate corresponding units.

$$= \frac{136 \cancel{\text{ in.}}}{1} \times \frac{1 \text{ ft.}}{12 \cancel{\text{ in.}}}$$

$$= \frac{136}{12} \text{ ft.}$$

$$= 11\frac{4}{12} \text{ ft.}$$

Since this measurement is equal to the measurement in *Example 2*, the conversion is verified.

Example 3 Solving a Problem Involving Two Unit Conversions

The school council has 6 yd. of fabric that will be cut into strips 5 in. wide to make decorative banners for the school dance.

a) How many banners can be made?

b) Use unit analysis to verify the conversions.

SOLUTION

a) Since the width of a banner is measured in inches, convert the length of the material to inches.
Convert 6 yd. to inches.

1 yd. = 3 ft.

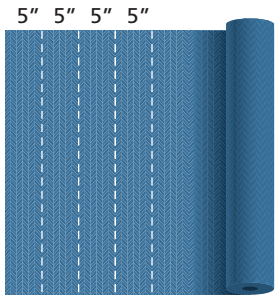
6 yd. = 6(3 ft.)

6 yd. = 18 ft.

1 ft. = 12 in.

18 ft. = 18(12 in.)

18 ft. = 216 in.



(Solution continues.)

CHECK YOUR UNDERSTANDING

3. Tyrell has 4 yd. of cord to make friendship bracelets. Each bracelet needs 8 in. of cord.

a) How many bracelets can Tyrell make?

b) Use unit analysis to check the conversions.

[Answer: a) 18]

The number of banners is: $\frac{216}{5} = 43.2$

43 banners can be made.

b) To convert yards to inches, first convert yards to feet, then convert feet to inches.

Write a conversion factor for yards and feet, with feet in the numerator: $\frac{3 \text{ ft.}}{1 \text{ yd.}}$

Write a conversion factor for feet and inches, with inches in the numerator: $\frac{12 \text{ in.}}{1 \text{ ft.}}$

$$\text{Then, } 6 \text{ yd.} \times \frac{3 \text{ ft.}}{1 \text{ yd.}} \times \frac{12 \text{ in.}}{1 \text{ ft.}} = \frac{6 \cancel{\text{yd.}}}{1} \times \frac{3 \cancel{\text{ft.}}}{1 \cancel{\text{yd.}}} \times \frac{12 \text{ in.}}{1 \cancel{\text{ft.}}}$$

$$= (6 \times 3 \times 12) \text{ in.}$$

$$= 216 \text{ in.}$$

Since this measurement is equal to the measurement in part a, the conversion is verified.

What conversion factor could you use to convert the units in one step?

Example 4 Solving a Problem Involving Scale Diagrams

A map of Alaska has a scale of 1:4 750 000. The distance on the map between Paxson and the Canadian border is $3\frac{11}{16}$ in. What is this distance to the nearest mile?

SOLUTION

The map scale is 1 in. represents 4 750 000 in.

$3\frac{11}{16}$ in. represents

$3\frac{11}{16}(4\,750\,000 \text{ in.}) = 17\,515\,625 \text{ in.}$

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(3+11/16)*4750000
                17515625
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Divide by 12 to convert 17 515 625 in. to feet:

$\frac{17\,515\,625}{12} = 1\,459\,635.417\ldots$

Divide by 5280 to convert 1 459 635.417… ft. to miles:

$\frac{1\,459\,635.417\ldots}{5280} = 276.446\ldots$

The distance between Paxson and the Canadian border is approximately 276 mi.

CHECK YOUR UNDERSTANDING

4. On the map with a scale of 1:4 750 000, the distance between Seward and Anchorage in Alaska is $1\frac{3}{4}$ in. What is the distance between these two towns to the nearest mile?

[Answer: approximately 131 mi.]

Discuss the Ideas

1. Why is it important to measure with standard units? Give an example.
2. When might you use referents to estimate a measure?

Exercises

A

3. Which imperial unit is the most appropriate unit to measure each item? Justify your choice.
- **a)** the height of your desk
- **b)** the thickness of a mattress
- **c)** the width of a car
- **d)** the length of a flat panel TV
- **e)** the distance from the school to your home

4. **a)** Which imperial unit is most appropriate to measure the length of a piece of notepaper? Why?
b) Use a referent to estimate the length of a piece of notepaper. Explain the process. Measure to check your estimate.

5. **a)** Which imperial unit is most appropriate to measure the height of the door in your classroom? Why?
b) Use a referent to estimate the height of the door. Explain the process. Measure to check your estimate.

6. Estimate each measurement in imperial units.
- **a)** the length of your arm from wrist to elbow
- **b)** the height of your classroom
- **c)** the distance from your classroom to the school office
- **d)** the perimeter of your school grounds

7. Convert:
- **a)** 3 ft. to inches
- **b)** 63 yd. to feet
- **c)** 48 in. to feet

B

8. Convert:
- **a)** 2 mi. to feet
- **b)** 574 in. to yards, feet, and inches
- **c)** 7390 ft. to miles, yards, and feet

9. Explain how to convert a measurement of 165 in. to a measurement in yards, feet, and inches.

10. Carolyn is building a pen for her dog. The perimeter of the pen is 52 ft.
a) Explain how you can use a proportion to convert the perimeter to yards and feet.
b) The fencing material is sold by the yard. It costs $10.99/yd. What is the cost of material before taxes?

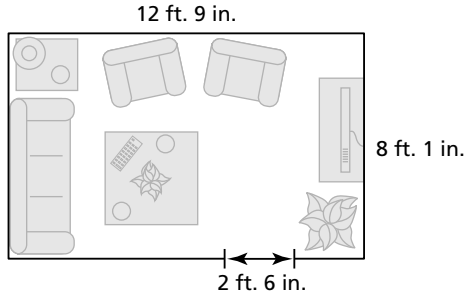
11. David has 10 yd. of material that he will cut into strips 15 in. wide to make mats.
a) How many mats can David make?
b) Use unit analysis to verify the conversions.

12. Pierre-Marc converted 21 ft. 9 in. into yards, feet, and inches. His answer was 7 yd. 1 ft. 6 in. Is his answer correct? If your answer is no, show the correct conversion.

13. In 2008, Sandy Allen and Leonid Stadnyk were the world's tallest living woman and man. Their respective heights are 7 ft. 7 in. and 8 ft. 5 in. How many inches shorter is Sandy than Leonid?

14. A wallpaper border is to be pasted halfway up the wall around a child's bedroom.

a) What is the total length of border needed?

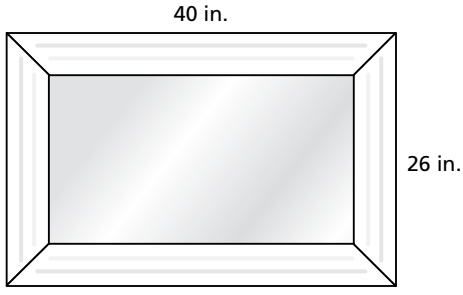


b) The border is purchased in 12-ft. rolls. How many rolls are required?

c) Each roll of border costs $12.49. How much will the border cost, before taxes?

15. **a)** In a basement renovation, the contractor measured the length of a wall in a square room as 18 ft. 4 in. The width of the doorway is 3 ft. The contractor plans to place wood trim along the bottom of each wall. The trim costs $1.69/ft. What is the cost of the trim for the room, before taxes?

b) The contractor uses the same trim around this window.



Determine the cost of the trim for the window, before taxes.

16. A 3-D puzzle of the Eiffel Tower has a scale of 1:360. In the puzzle, the tower is $35\frac{2}{5}$ in. tall. What is the height of the Eiffel Tower in feet?

17. A map of Québec has a scale of 1:1 500 000. On the map, the distance between Trois-Rivières and Québec City is $2\frac{5}{8}$ in. What is the distance between these cities to the nearest mile?

18. A gardener recommends planting tulip bulbs 8 in. apart. Erica follows the gardener's advice and plants tulips beside her 18-ft. sidewalk. How many tulip bulbs will Erica need?

C

19. A student can walk 30 ft. in 10 s. How far could she walk in 1 h? Write the answer in miles and yards.

20. On a map of British Columbia and Alaska, the distance between Prince Rupert, B.C., and Ketchikan, AK, is $2\frac{9}{16}$ in. The actual distance between these towns is 95 mi. What is the map scale, to the nearest thousand?

21. Twenty reams of paper form a stack 40 in. high. Each ream costs $3.

a) What is the value of a stack that has the same height as Mount Logan, which is 19 500 ft. high?

b) How can you use mental math and estimation to determine if your answer is reasonable?

22. Five toonies form a stack that is $\frac{2}{5}$ in. high. What is the approximate value of a stack of toonies that spans the 100 mi. between Plamondon and Bonnyville, AB?

Reflect

How would you use a referent to estimate a length? How do you determine the appropriate imperial unit to record the measure of that length?

MATH LAB

1.2 Measuring Length and Distance

LESSON FOCUS

Use measuring instruments and personal strategies to determine linear measures.

Make Connections

How do you think the person who makes these glass vases checks that the dimensions of each vase are correct?

Construct Understanding

TRY THIS

Work with a partner.

You will need:

- rulers and measuring tapes in both SI and imperial measures, metre stick, and yard stick
- string
- calipers
- 3 objects with different sizes and shapes, such as a can, a bottle, an eraser, a paper clip, a sunflower seed, a desk

How can you measure the perimeter of a can without using the formula for the circumference of a circle?

Repeat the following steps for each object:

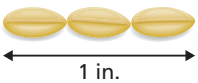
A. Sketch each object.
Use a referent to estimate all possible linear measures of the object in:

- imperial units
- SI units

Record these estimates on the sketch.
Justify your choice of units.

B. Choose appropriate measuring instruments. Use both imperial units and SI units to measure the object in as many ways as you can. Record the measurements on the sketch.

C. Describe any problems you had making the measurements and the strategies you used to solve the problems.



THE WORLD OF MATH

Historical Moment: The Origin of the Inch

The word *inch* may be derived from the Latin word *uncia* meaning "one-twelfth part," referring to one-twelfth the length of a man's foot. Alternatively, the Anglo-Saxon term *inch* was defined as the length of 3 corns of barley. King David I of Scotland described an *ynche* as the width of a man's thumb at the base of the nail; this is supported by the similarity of the word "inch" to the word "thumb" in several languages. For example, in French, *pouce* means inch and also means thumb. In Swedish, *tum* is inch and *tumme* is thumb.

Assess Your Understanding

1. Describe a referent you could use for each SI measure.

a) 1 m **b)** 1 cm **c)** 1 mm

2. What referent could you use for 1 km?

3. What are the limitations of the use of calipers?

4. Describe a strategy you would use to estimate and then measure each length.

a) the circumference of a cylindrical garbage can
b) the thickness of your hand
c) the distance between your home and the closest store
d) the distance between two cities on a map
e) the distance around an oval running track
f) the length of an eyelash

5. What referents would you use to estimate the length, in both SI units and imperial units, of the Lion's Gate Bridge in Vancouver, B.C.? Explain how you could measure the length in both units.

6. Look around the classroom.

a) Identify objects that were built using imperial measures. Justify your choice.
b) Identify objects that were built using SI measures. Justify your choice.

1.3 Relating SI and Imperial Units

LESSON FOCUS

Convert measurements between SI units and imperial units.

The driver of this solar powered car was at a Canada/U.S. border crossing during the North American Solar Challenge in July 2005.

Make Connections

Two cars are driven in opposite directions from a Canada/United States border crossing.
In one hour, Hana drove 62 mi. south while Farrin drove 98 km north.
How could you determine which vehicle travelled farther from the border?

Construct Understanding

TRY THIS

Work on your own.

You will need a ruler, metre stick, or measuring tape that has both SI and imperial units.

A. Look at the SI and imperial scales on the ruler.
Estimate the length of 1 in. to the nearest tenth of a centimetre.
Estimate the length of 1 cm to the nearest fraction of an inch.

B. Copy this table. Use the results from Step A to complete the table. In the first column, write the numbers as decimals to the nearest tenth. In the second column, write the numbers as fractions or mixed numbers.

Imperial Units to SI Units	SI Units to Imperial Units
1 in. ≐ cm	1 cm ≐ in.
1 ft. ≐ cm	1 m ≐ in. 1 m ≐ ft. in. 1 m ≐ yd. ft. in.
1 yd. ≐ cm 1 yd. ≐ m	1 km ≐ yd.
1 mi. ≐ m	

C. Choose 3 different objects in the classroom. For each object:

- Measure its length in SI units.
- Use the completed table in Step B to write the measurement in imperial units.
- Check your answer by measuring the length in imperial units.

D. Choose 3 more objects in the classroom. For each object:

- Measure its length in imperial units.
- Use the completed table in Step B to write the measurement in SI units.
- Check your answer by measuring the length in SI units.

Each measurement in the imperial system relates to a corresponding measurement in the SI system.

This table shows some approximate relationships between imperial units and SI units.

SI Units to Imperial Units	Imperial Units to SI Units
$1 \text{ mm} \doteq \frac{4}{100}$ in.	1 in. $\doteq$ 2.5 cm
$1 \text{ cm} \doteq \frac{4}{10}$ in.	1 ft. $\doteq$ 30 cm 1 ft. $\doteq$ 0.3 m
1 m $\doteq$ 39 in. $1 \text{ m} \doteq 3\frac{1}{4}$ ft.	1 yd. $\doteq$ 90 cm 1 yd. $\doteq$ 0.9 m
$1 \text{ km} \doteq \frac{6}{10}$ mi.	1 mi. $\doteq$ 1.6 km

Some conversions are exact; for example,
1 in. = 2.54 cm
1 yd. = 91.44 cm

We can use the data in the table above to convert between SI and imperial units of measure.

Example 1 Converting from Metres to Feet

A bowling lane is approximately 19 m long.

What is this measurement to the nearest foot?

SOLUTION

From the table, $1 \text{ m} \doteq 3\frac{1}{4}$ ft.

So, $19 \text{ m} \doteq 19(3\frac{1}{4} \text{ ft.})$

$$19 \text{ m} \doteq 19\left(\frac{13}{4}\right)\text{ft.}$$

$$19 \text{ m} \doteq \frac{247}{4} \text{ ft.}$$

$$19 \text{ m} \doteq 61\frac{3}{4} \text{ ft.}$$

A length of 19 m is approximately 62 ft.

CHECK YOUR UNDERSTANDING

1. A Canadian football field is approximately 59 m wide.

What is this measurement to the nearest foot?

[Answer: approximately 192 ft.]

Example 2 Converting between Miles and Kilometres

After meeting in Emerson, Manitoba, Hana drove 62 mi. south and Farrin drove 98 km north. Who drove farther?

SOLUTIONS

To compare the distances, convert one measurement so the units are the same.

Method 1

Convert the distance Hana drove from miles to kilometres.

$$1 \text{ mi.} \doteq 1.6 \text{ km}$$

So, $62 \text{ mi.} \doteq 62(1.6 \text{ km})$

$$62 \text{ mi.} \doteq 99.2 \text{ km}$$

Since $99.2 \text{ km} > 98 \text{ km}$, Hana drove farther.

Method 2

Convert the distance Farrin drove from kilometres to miles.

$$1 \text{ km} \doteq \frac{6}{10} \text{ mi.}$$

So, $98 \text{ km} \doteq 98\left(\frac{6}{10} \text{ mi.}\right)$

$$98 \text{ km} \doteq 58\frac{8}{10} \text{ mi., or } 58\frac{4}{5} \text{ mi.}$$

Since $58\frac{4}{5} < 62$, Hana drove farther.

CHECK YOUR UNDERSTANDING

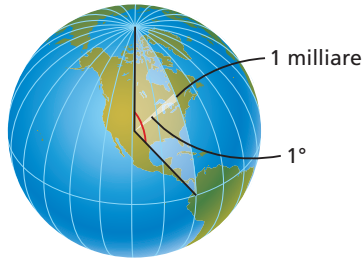
2. After meeting in Osoyoos, B. C., Takoda drove 114 km north and Winona drove 68 mi. south. Who drove farther?

[Answer: Takoda]

THE WORLD OF MATH

Historical Moment: The Decimal System of Measurement

Gabriel Mouton, who lived in Lyons, France from 1618 – 1694, was the first person to propose a decimal system of measurement. In 1670, he created a unit equal to the length of 1° of longitude on Earth's surface. Mouton suggested that this unit, the *milliare*, be successively further divided by 10 to produce these units: *centuria*, *decuria*, *virga*, *virgula*, *decima*, *centesima*, and *millesima*.



Mouton's ideas were studied at the time but it was more than 100 years later that the French Academy of Sciences adopted a decimal system of measurement. Mouton's milliare corresponds to 1 nautical mile that is used in the shipping and aviation industries today.

Example 3 Solving a Problem that Involves Unit Conversions

Alex is 6 ft. 2 in. tall. To list his height on his driver's license application, Alex needs to convert this measurement to centimetres.

a) What is Alex's height to the nearest centimetre?

b) Use mental math and estimation to justify that the answer is reasonable.

SOLUTION

a) Convert 6 ft. 2 in. to inches.

$$1 \text{ ft.} = 12 \text{ in.}$$

$$\text{So, } 6 \text{ ft.} = 6(12 \text{ in.})$$

$$6 \text{ ft.} = 72 \text{ in.}$$

$$\text{And, } 6 \text{ ft. } 2 \text{ in.} = 72 \text{ in.} + 2 \text{ in.}$$

$$6 \text{ ft. } 2 \text{ in.} = 74 \text{ in.}$$

Write a proportion to convert 74 in. to centimetres.

Let h represent Alex's height in centimetres.

The ratio of h centimetres to 74 in. is approximately equal to the ratio of 1 cm to $\frac{4}{10}$ in.

Write $\frac{4}{10}$ as 0.4.

$$\frac{h}{74} \doteq \frac{1}{0.4}$$

Multiply each side by 74.

$$74\left(\frac{h}{74}\right) \doteq 74\left(\frac{1}{0.4}\right)$$

$$h \doteq \frac{74}{0.4}$$

$$h \doteq 185$$

Alex is approximately 185 cm tall.

b) To check:

$$1 \text{ ft.} \doteq 30 \text{ cm}$$

$$6 \text{ ft.} \doteq 180 \text{ cm}$$

So, 6 ft. 2 in. $\doteq$ 185 cm is reasonable.

CHECK YOUR UNDERSTANDING

3. Nora knows that she is 5 ft. 7 in. tall.

a) What height in centimetres will she list on her driver's license application?

b) Use mental math and estimation to justify that the answer is reasonable.

[Answer: a) 168 cm]

What other strategy could you use to determine Alex's height in centimetres?

Example 4 Estimating and Calculating Using Unit Conversions

A truck driver knows that her semitrailer is 3.5 m high. The support beams of a bridge are 11 ft. 9 in. high. Will the vehicle fit under the bridge? Justify the answer.

SOLUTION

Write a proportion to convert 3.5 m to feet.

Let h represent the height of the vehicle in feet.

The ratio of h feet to 3.5 m is approximately equal to the ratio of 1 ft. to 0.3 m.

$$\frac{h}{3.5} \doteq \frac{1}{0.3}$$ Multiply each side by 3.5.

$$3.5\left(\frac{h}{3.5}\right) \doteq 3.5\left(\frac{1}{0.3}\right)$$

$$h \doteq \frac{3.5}{0.3}$$

$$h \doteq 11.\overline{6}, \text{ or } 11\frac{2}{3}$$

The vehicle is approximately $11\frac{2}{3}$ ft., or 11 ft. 8 in. high; so it should fit under the bridge.

This height is an estimate that is very close to the bridge height. To be sure the vehicle will fit, calculate an exact conversion. Convert the height of the vehicle in centimetres to inches.

Use the conversion: 2.54 cm = 1 in.
So,

$350 \text{ cm} = \frac{350}{2.54} \text{ in.}$ Converting 3.5 m to 350 cm

$350 \text{ cm} = 137.7952\ldots \text{ in.}$ Convert inches to feet.

$350 \text{ cm} = \frac{137.7952\ldots}{12} \text{ ft.}$

$350 \text{ cm} = 11.4829\ldots \text{ ft.}$

This measurement is a little less than $11\frac{1}{2}$ ft. or 11 ft. 6 in., so the vehicle will fit under the bridge.

CHECK YOUR UNDERSTANDING

4. A truck driver knows that his load is 15 ft. wide. Regulations along his route state that any load over 4.3 m wide must have wide-load markers and an escort with flashing lights. Does this vehicle need wide-load markers? Justify your answer.

[Answer: Yes, the load is approximately 4.6 m wide.]

Discuss the Ideas

1. When might you want to convert:
   - a measurement in SI units to imperial units?
   - a measurement in imperial units to SI units?
2. What relationships can help you check that an answer is reasonable when you convert between systems of measurement?
3. When you use unit analysis to verify an answer, how do you decide which conversion factor to use?

Exercises

A

4. Convert each measurement. Answer to the nearest tenth.
- **a)** 16 in. to centimetres
- **b)** 4 ft. to metres
- **c)** 5 yd. to metres
- **d)** 1650 yd. to kilometres
- **e)** 6 mi. to kilometres
- **f)** 2 in. to millimetres

5. Convert each measurement.
- **a)** 25 mm to the nearest inch
- **b)** 2.5 m to the nearest foot
- **c)** 10 m to the nearest yard
- **d)** 150 km to the nearest mile

6. Convert each measurement. Answer to the nearest tenth.
- **a)** 1 ft. 10 in. to centimetres
- **b)** 2 yd. 2 ft. 5 in. to centimetres
- **c)** 10 yd. 1 ft. 7 in. to metres

B

7. **a)** Convert each measurement.
- **i)** 75 cm to feet and the nearest inch
- **ii)** 274 cm to yards, feet, and the nearest inch
- **iii)** 10 000 m to the nearest mile

b) Use mental math and estimation to justify that each answer in part a is reasonable.

8. The dimensions of a lacrosse field are 110 yd. by 60 yd. What are these dimensions to the nearest tenth of a metre?

9. The Fraser River is approximately 1375 km long. The Tennessee River is approximately 886 mi. long. Which river is longer? Justify your answer.

10. On a road trip in Montana, USA, Elise sees this road sign:

Elise tests the accuracy of her car's odometer and tracks the distance she drove from that sign to Helena's city limits. Her odometer showed 142 km. Is the odometer accurate? Explain.

11. A retail fabric store advertises a storewide sale. It lists a certain material for $0.89/yd. A fabric warehouse is selling the same material for $0.93/m.
- **a)** Which store has the better price?
- **b)** Use mental math and estimation to justify that the answer is reasonable.

12. In preparation for les Jeux de la francophonie canadienne, Jean-Luc ran two laps around a 400-yd. track. Michael did seven 110-m hurdle practice races.
- **a)** Who ran farther?
- **b)** Use unit analysis to verify the conversion.

13. The tallest structure in Canada is the CN Tower in Toronto. It is 553.3 m tall. The tallest structure in the United States is the Willis Tower, previously known as the Sears Tower, in Chicago. It is 1451 ft. tall.

a) Determine the height of the CN Tower in feet and the height of the Willis Tower in metres.

b) Which structure is taller? Explain how you know.

c) Determine the difference in the heights of the structures, in metres and to the nearest foot.

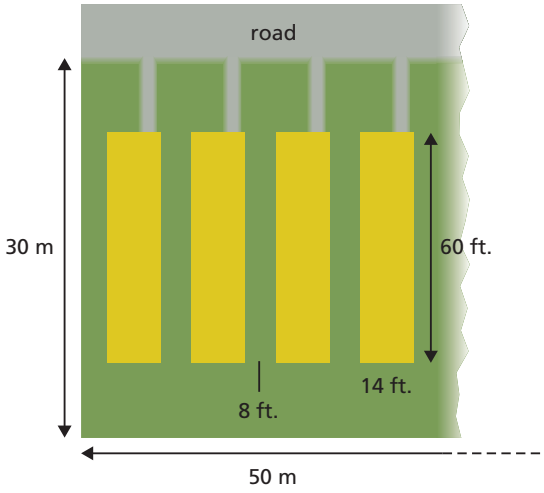
14. On a lease site, an oil company determined that there was an oil reserve 1400 m beneath the surface. While the crew drilled, it lined the hole with casing. Each 32-ft. piece of casing was welded to the previous piece to prevent the hole from collapsing. How many sections of casing did the crew need to reach the oil reserve?

C

15. The rim of a basketball net is mounted 10 ft. off the ground. A basketball player has a maximum reach of 2.5 m. How high, in inches, does the player need to jump to reach 6 in. above the rim?

16. An electrician was hired to run the wires for a surround-sound stereo speaker system. She purchased 2 rolls of 14-gauge speaker wire. Each roll contains 4 m of wire. For each of 2 speakers, 2 ft. of wire are required. For each of the other 2 speakers, 11 ft. of wire are required. Will the electrician have enough wire? If your answer is no, what length of wire in centimetres will she need? If your answer is yes, what length of wire in centimetres will be left over?

17. A real-estate developer purchased a 30-m by 50-m plot of land to create a mobile home park. The developer sketched this plan:



What is the maximum number of homes the developer can fit on this land?

18. The imperial unit to measure an area of land is the *acre*. During the initial agricultural expansion of the western provinces, the Canadian government offered 160 acres of land free to settlers who were willing to immigrate to Canada. Today, Canada uses the *hectare* to measure land area:

1 hectare $\doteq$ 2.471 acres

a) How many hectares did each settler receive?

b) One hundred sixty acres is a square with a side length of one-half a mile. How many hectares are in one square mile?

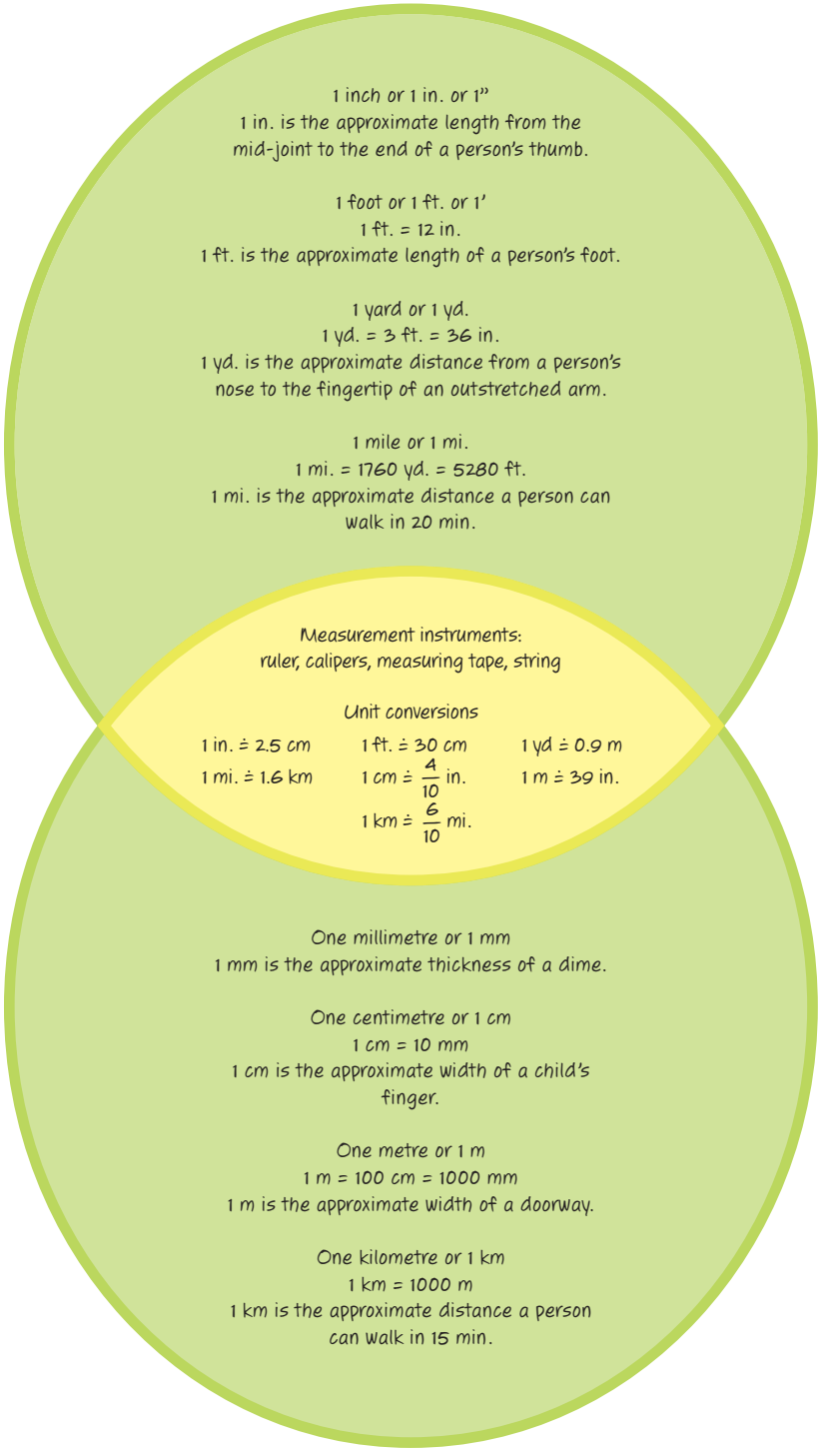
Reflect

What strategies do you know for converting a measure in imperial units to a measure in SI units? Include examples in your explanation.

CHECKPOINT 1

Connections

Imperial Units of Length



SI Units of Length

Concept Development

- **In Lesson 1.1**
  - You developed **referents** for units of linear measure in the **imperial system**.
  - You **converted between units** within the imperial system.
  - You used conversion factors and **proportional reasoning** to solve problems.
  - You used **unit analysis** to verify conversions.
- **In Lesson 1.2**
  - You **estimated and measured** dimensions of objects using both the **imperial** and **SI systems.**
  - You developed **personal strategies** and used measuring instruments to **determine linear measurements.**
- **In Lesson 1.3**
  - You established approximate **relationships between SI and imperial units** of length.
  - You used conversion factors and **proportional reasoning** to solve problems.
  - You used **unit analysis** to verify conversions.

Assess Your Understanding

1.1

1. Use imperial units to estimate each measure. State the referent you used in each case.
a) the distance from your elbow to your wrist
b) the length of a cell phone
c) the width of a twin bed
d) the height of the ceiling in the gymnasium

2. Choose the most appropriate imperial unit, then measure the items in question 1 parts a and b.

3. Convert:
a) 80 ft. to yards and feet
b) 3 mi. to yards
c) 2 yd. 1 ft. to inches

4. Lindsay is 51 in. tall. Sidney is 3 ft. 11 in. tall.
Who is shorter? How do you know?

1.2

5. Measure each item in both SI and imperial units:
a) the perimeter of your classroom
b) the width of a car
c) the circumference of a dinner plate

6. Describe how you would determine the diameter of a marble. State the measuring instrument and the unit you would use.

1.3

7. Convert each measurement.
a) 13 m to yards and the nearest foot
b) 4 ft. to the nearest centimetre
c) 2 km to miles and the nearest yard
d) 25 000 cm to yards, feet, and the nearest inch
e) 13 000 in. to the nearest tenth of a metre
f) 1750 mm to feet and the nearest inch

8. A thin strip of wood laminate is to be glued to the edges of a table. The length of laminate required is equal to the perimeter of the table, which has dimensions 30 cm by 115 cm. The laminate can only be bought in lengths of whole numbers of feet. What length of laminate is needed?

1.4 Surface Areas of Right Pyramids and Right Cones

LESSON FOCUS

Solve problems involving the surface areas of right pyramids and right cones.

Make Connections

The ancient pyramids at Giza, Egypt, were built about 4500 years ago.

The pyramid above has a square base with a side length of 755 feet. The original height of the pyramid was 481 feet. Archeologists believe that the pyramid was once covered with a white limestone casing. How could you calculate the area that was once covered with limestone?

Construct Understanding

TRY THIS

Work in a group.

You will need a right pyramid and a ruler with SI units and imperial units.
You may need 1-cm grid paper and 1-in. grid paper.
One-half of the group will use imperial units and the other half will use SI units.

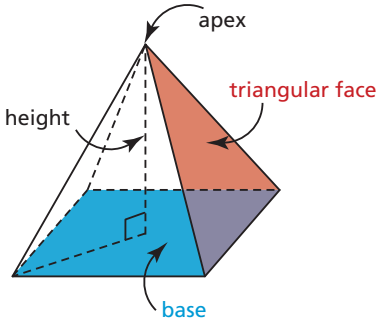
A. Estimate the surface area of the pyramid, including its base. Describe your strategy for estimating.

B. Determine the surface area of the pyramid. Describe the measurements you made and the strategies you used.

C. Compare your strategies with those of another group that measured a different type of pyramid. Did the strategy depend on the type of pyramid? Explain.

We use exponents when we write units for area; for example, five square inches is 5 $in.^2$ and five square centimetres is 5 cm^2.

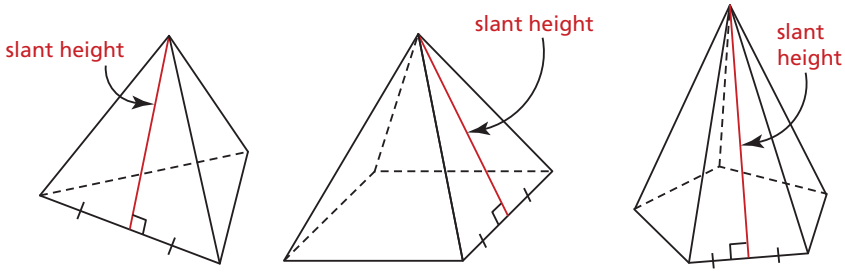
A **right pyramid** is a 3-dimensional object that has triangular faces and a base that is a polygon. The shape of the base determines the name of the pyramid. The triangular faces meet at a point called the **apex**. The *height* of the pyramid is the perpendicular distance from the apex to the centre of the base.



When the base of a right pyramid is a regular polygon, the triangular faces are congruent. Then the **slant height** of the right pyramid is the height of a triangular face.

A *tetrahedron* is a triangular pyramid.

A *regular tetrahedron* has 4 congruent equilateral triangular faces.



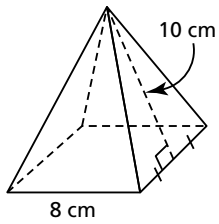
regular tetrahedron

right square pyramid

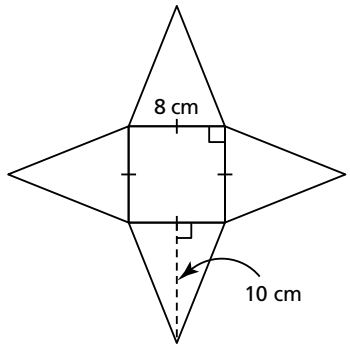
right pentagonal pyramid

The surface area of a right pyramid is the sum of the areas of the triangular faces and the base.

This right square pyramid has a slant height of 10 cm and a base side length of 8 cm.



This net shows the faces and base of the pyramid.



The area, A, of each triangular face is:

$A = \frac{1}{2}(8)(10)$

$A = 40$

The area, B, of the base is:

$B = (8)(8)$

$B = 64$

So, the surface area, SA, of the pyramid is:

$SA = 4A + B$

$SA = 4(40) + 64$

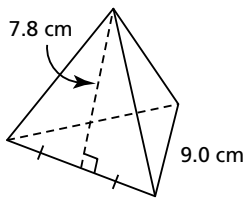
$SA = 224$

The surface area of the pyramid is 224 cm^2.

> The area, A, of a triangle with base, b, and height, h, is:
> $A = \frac{1}{2}bh$

Example 1 Determining the Surface Area of a Regular Tetrahedron Given Its Slant Height

Jeanne-Marie measured then recorded the lengths of the edges and slant height of this regular tetrahedron. What is its surface area to the nearest square centimetre?



SOLUTION

The regular tetrahedron has 4 congruent faces. Each face is a triangle with base 9.0 cm and height 7.8 cm.

The area, A, of each face is:

$A = \frac{1}{2}(9.0 \text{ cm})(7.8 \text{ cm})$

The surface area, SA, is:

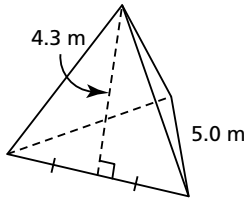
$SA = 4\left(\frac{1}{2}\right)(9.0 \text{ cm})(7.8 \text{ cm})$

$SA = 140.4 \text{ cm}^2$

The surface area of the tetrahedron is approximately 140 cm^2.

CHECK YOUR UNDERSTANDING

1. Calculate the surface area of this regular tetrahedron to the nearest square metre.



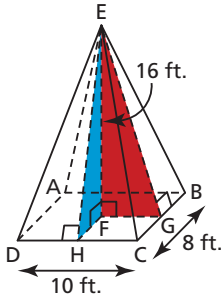
[Answer: 43 m^2]

Example 2 Determining the Surface Area of a Right Rectangular Pyramid

A right rectangular pyramid has base dimensions 8 ft. by 10 ft., and a height of 16 ft. Calculate the surface area of the pyramid to the nearest square foot.

SOLUTION

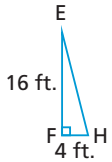
There are 4 triangular faces and a rectangular base. Sketch the pyramid and label its vertices. Opposite triangular faces are congruent. Draw the heights on two adjacent triangles.



In $\triangle EFH$, FH is $\frac{1}{2}$ the length of BC, so FH is 4 ft.

EF is the height of the pyramid, which is 16 ft.

Use the Pythagorean Theorem in right $\triangle EFH$.



$EH^2 = EF^2 + FH^2$

$EH^2 = 16^2 + 4^2$

$EH^2 = 272$

$EH = \sqrt{272}$

Area, A, of $\triangle EDC$ is:

$A = \frac{1}{2}(10)(\sqrt{272})$

$A = 5(\sqrt{272})$

Since $\triangle EDC$ and $\triangle EAB$ are congruent, the area of $\triangle EAB$ is $5(\sqrt{272})$.

In $\triangle EFG$, FG is $\frac{1}{2}$ the length of DC, so FG is 5 ft.

Use the Pythagorean Theorem in right $\triangle EFG$.

$GE^2 = EF^2 + FG^2$

$GE^2 = 16^2 + 5^2$

$GE^2 = 281$

$GE = \sqrt{281}$

Area, A, of $\triangle EBC$ is:

$A = \frac{1}{2}(8)(\sqrt{281})$

$A = 4(\sqrt{281})$

(Solution continues.)

CHECK YOUR UNDERSTANDING

2. A right rectangular pyramid has base dimensions 4 m by 6 m, and a height of 8 m. Calculate the surface area of the pyramid to the nearest square metre.

[Answer: approximately 108 m^2]

What is an advantage of using $EH = \sqrt{272}$ and $GE = \sqrt{281}$, instead of writing these square roots as decimals?

Since $\triangle EBC$ and $\triangle EAD$ are congruent, the area of $\triangle EAD$ is $4(\sqrt{281})$.

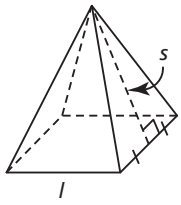
Area, B, of the base of the pyramid is:
$B = (8)(10)$
$B = 80$

Each of two triangles has area $5(\sqrt{272})$, and each of the other two triangles has area $4(\sqrt{281})$.

Surface area, SA, of the pyramid is:
$SA = 2(5)(\sqrt{272}) + 2(4)(\sqrt{281}) + 80$

$SA = 379.0286\ldots$
The surface area of the pyramid is approximately 379 square feet.

We can determine a formula for the surface area of any right pyramid with a regular polygon base. Consider this right square pyramid. Each triangular face has base ℓ and height s.



The area, A, of each triangular face is:
$A = \frac{1}{2}$(base)(height)
$A = \frac{1}{2}\ell s$

So, the area of the 4 triangular faces is:
$4\left(\frac{1}{2}\ell s\right) = 4\left(\frac{1}{2}\right)\ell s$ Rearrange the numbers and variables.

$= \left(\frac{1}{2}s\right)(4\ell)$

The area of the triangular faces of a pyramid is called the **lateral area**, denoted A_L.

We use A_L to represent the lateral area so it is not confused with A, which represents the area of a triangular face.

$A_L = \left(\frac{1}{2}s\right)(4\ell)$

The term 4ℓ is the perimeter of the base of the pyramid, so

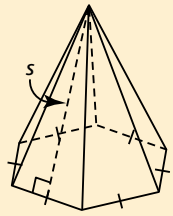
Surface area of the pyramid $= (\frac{1}{2}$ slant height)(perimeter of base) + base area

Since any right pyramid with a regular polygon base has congruent triangular faces, the same formula is true for any of these pyramids.

Surface Area of a Right Pyramid with a Regular Polygon Base

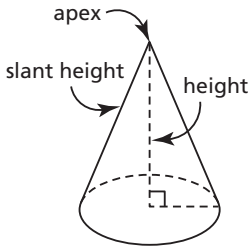
For a right pyramid with a regular polygon base and slant height s,

$$\text{Surface area} = \frac{1}{2}s(\text{perimeter of base}) + (\text{base area})$$

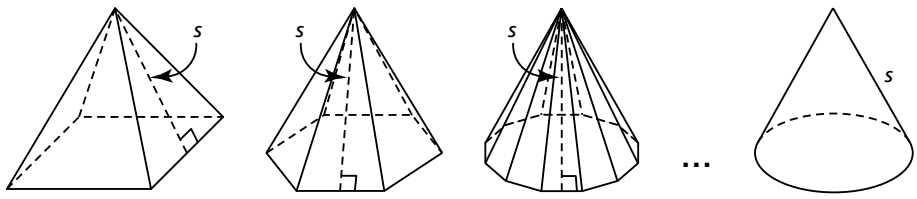


A *right circular cone* is a 3-dimensional object that has a circular base and a curved surface. The *height* of the cone is the perpendicular distance from the apex to the base. The *slant height* of the cone is the shortest distance on the curved surface between the apex and a point on the circumference of the base.

A right circular cone is usually called a **right cone**.



One way to derive a formula for the surface area of a right cone is to consider the surface area of a right pyramid with a regular polygon base. Visualize the pyramid as the number of sides of the base increases.



The number of triangular faces increases. The lateral area of the right pyramid approaches the lateral area of a right cone.

$$\text{Lateral area of a right pyramid} = \frac{1}{2}s(\text{perimeter of base})$$

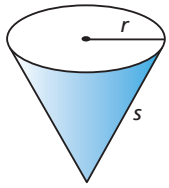
For a right cone, the perimeter of the base is the circumference of the circle. So, for a right cone with slant height s and base radius r:

$$\text{Lateral area} = \frac{1}{2}s(\text{perimeter of base})$$

$$\text{Lateral area} = \frac{1}{2}s(\text{circumference of base})$$

$$A_L = \frac{1}{2}s(2\pi r)$$

$$A_L = \pi rs$$

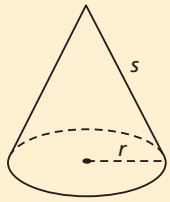


Surface Area of a Right Cone

Surface area = lateral area + base area

For a right cone with slant height s and base radius r:

$SA = \pi rs + \pi r^2$



Example 3 Determining the Surface Area of a Right Cone

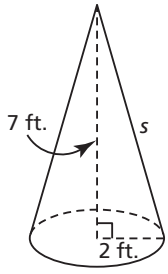
A right cone has a base radius of 2 ft. and a height of 7 ft. Calculate the surface area of this cone to the nearest square foot.

SOLUTION

Sketch a diagram.

Let s represent the slant height.

Visualize cutting the cone in half through a diameter of its base. This produces an isosceles triangle with a base that is equal to the diameter of the cone and a height that is equal to the height of the cone.



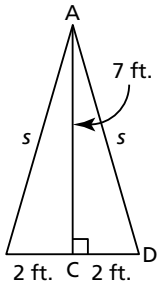
Use the Pythagorean Theorem in right $\triangle ACD$.

$s^2 = 7^2 + 2^2$

$s^2 = 49 + 4$

$s^2 = 53$

$s = \sqrt{53}$



Use the formula for the surface area of a right cone:

$SA = \pi rs + \pi r^2$

Substitute: $r = 2$, $s = \sqrt{53}$

$SA = \pi(2)(\sqrt{53}) + \pi(2)^2$

$SA = 58.3086\ldots$

The surface area of the cone is approximately 58 square feet.

CHECK YOUR UNDERSTANDING

3. A right cone has a base radius of 4 m and a height of 10 m. Calculate the surface area of this cone to the nearest square metre.

[Answer: approximately 186 m^2]

In *Example 3*, which part of the formula represents the lateral area of the cone? How do you know?

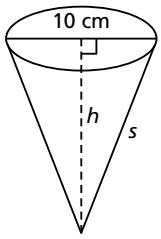
We can use the formula for surface area to determine unknown measurements.

Example 4 Determining an Unknown Measurement

The lateral area of a cone is 220 cm^2. The diameter of the cone is 10 cm. Determine the height of the cone to the nearest tenth of a centimetre.

SOLUTION

Sketch a diagram.



Let h represent the height of the cone and s the slant height.

The radius of the cone is 5 cm.

Use the formula for the lateral area, A_L, of the cone and solve for s.

$A_L = \pi rs$ Substitute: $A_L = 220$ and $r = 5$

$220 = \pi(5)s$ Divide both sides of the equation by 5π.

$\frac{220}{5\pi} = \frac{5\pi s}{5\pi}$

$s = \frac{220}{5\pi}$

$s = 14.0056\ldots$

To determine the height of the cone, use the Pythagorean Theorem in right $\triangle ABC$.

$5^2 + h^2 = s^2$ Substitute for s.

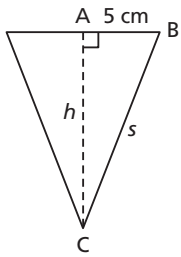
$25 + h^2 = (14.0056\ldots)^2$ Solve for h^2.

$h^2 = 196.1578\ldots - 25$

$h^2 = 171.1578\ldots$ Solve for h.

$h = \sqrt{171.1578\ldots}$

$h = 13.0827\ldots$



The height of the cone is approximately 13.1 cm.

CHECK YOUR UNDERSTANDING

4. A model of the Great Pyramid of Giza is constructed for a museum display. The surface area of the triangular faces is 3000 square inches. The side length of the base is 50 in. Determine the height of the model to a tenth of an inch.

[Answer: approximately $16\frac{3}{5}$ in.]

Discuss the Ideas

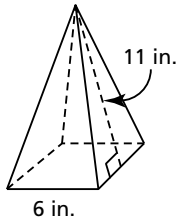
1. How do you determine the surface area of a right pyramid?
2. When you see a picture of a right pyramid with a regular polygon base, how do you identify its height and its slant height?
3. How is calculating the surface area of a right pyramid like calculating the surface area of a right cone? How is it different?

Exercises

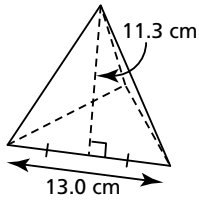
A

4. Determine the lateral area of each right pyramid to the nearest square unit.

a) square pyramid



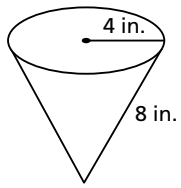
b) regular tetrahedron



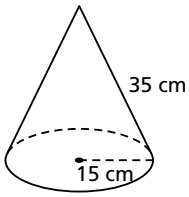
5. Determine the surface area of each right pyramid in question 4, to the nearest square unit.

6. Determine the lateral area of each right cone to the nearest square unit.

a)



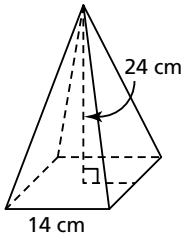
b)

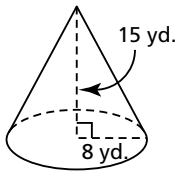


7. Determine the surface area of each right cone in question 6, to the nearest square unit.

8. Calculate the surface area of each object to the nearest square unit.

a) right square pyramid **b)** right cone





B

9. The slant height of a right square pyramid is 73 ft. and the side length of the base is 48 ft.

a) Sketch the pyramid.

b) Determine its lateral area to the nearest square foot.

10. The Great Pyramid at Giza has a square base with side length 755 ft. and an original height of 481 ft. Determine its original surface area to the nearest square foot.

11. Aiden built a cone-shaped volcano for a school science project. The volcano has a base diameter of 32 cm and a slant height of 45 cm.

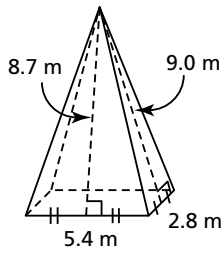
a) What is the lateral area of the volcano to the nearest tenth of a square centimetre?

b) The paint for the volcano's surface costs \$1.99/jar, and one jar of paint covers 400 cm^2. How much will the paint cost?

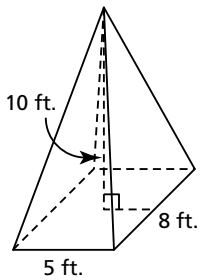
12. A road pylon approximates a right cone with perpendicular height 53 cm and base diameter 18 cm. The lateral surface of the pylon is to be painted with reflective paint. What is the area that will be painted? Answer to the nearest square centimetre.

13. Determine the surface area of each right rectangular pyramid to the nearest square unit.

a)



b)

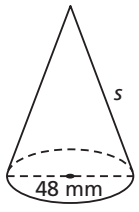


14. The Royal Saskatchewan Museum in Regina has a tipi in its First Nations Gallery. The tipi approximates a cone with a base diameter of 3.9 m and a height of 4.6 m. A Cree woman from Chitek Lake tanned, prepared, and sewed 15 bison hides to make the cover. To the nearest tenth of a square metre, what area did each bison hide cover? What assumptions did you make?

15. A farmer unloaded grain onto a tarp on the ground. The grain formed a cone-shaped pile that had a diameter of 12 ft. and a height of 8 ft. Determine the surface area of the exposed grain to the nearest square foot.

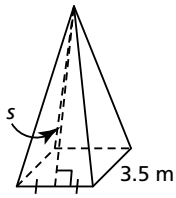
16. For each object, its surface area, *SA*, and some dimensions are given. Calculate the dimension indicated by the variable to the nearest tenth of a unit.

a) right cone



$SA = 7012 \text{ mm}^2$

b) right square pyramid



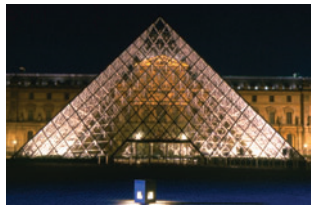
$SA = 65.5 \text{ m}^2$

17. A toy block manufacturer needs to cover its wooden blocks with a non-toxic paint.
One block is a right square pyramid with a base length of 2 in. and a slant height of $3\frac{1}{2}$ in.
A second block is a right cone that has a slant height of $3\frac{1}{2}$ in. and a base radius of 1 in.
A third block is a right rectangular prism with base dimensions 2 in. by 1 in. and a height of 3 in.

a) When the blocks rest on their bases, which block is tallest? How do you know?

b) Which block requires the most paint?

18. The Louvre art museum in Paris, France, has a glass square pyramid at its entrance. The side length of the base of the right pyramid is 35.0 m and its height is 20.6 m. The Muttart Conservatory in Edmonton, Alberta, has four right square pyramids also with glass faces. One of the largest pyramids has a base side length of 25.7 m and a height of 24.0 m.



Which pyramid requires more glass to enclose its space?

C

19. Determine the surface area of each right pyramid to the nearest tenth of a square unit.

a) a right pyramid with a base that is a regular hexagon with side length 5.5 cm; each triangular face has 2 equal sides with length 7.5 cm

b) a right pyramid with a base that is a regular pentagon with side length 2.4 m and the distance of each vertex from the centre of the base is 2.0 m; the height is 3.9 m

20. A right cone has a height of 8 ft. and a base circumference of 12 ft. Determine the surface area of the cone to the nearest square foot.

21. A right pyramid has a surface area of 258 cm^2. A right cone has a base radius of 4 cm. The cone and pyramid have equal surface areas. What is the height of the cone to the nearest tenth of a centimetre?

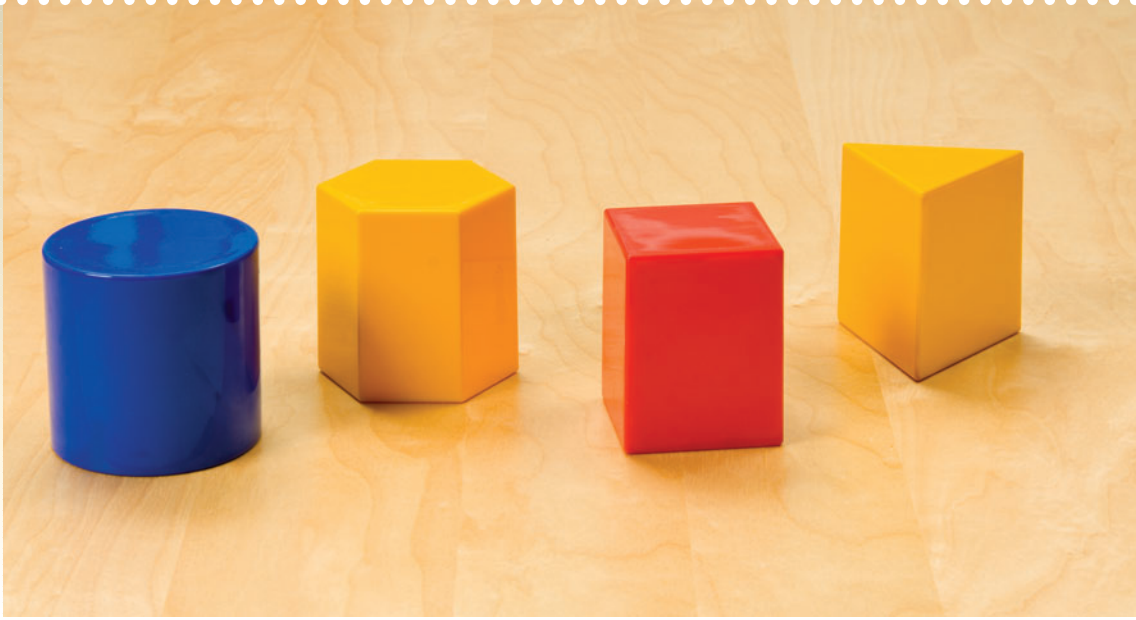
Reflect

What do you need to know to be able to calculate the surface areas of a right pyramid and a right cone? Include labelled diagrams in your explanation.

1.5 Volumes of Right Pyramids and Right Cones

LESSON FOCUS

Solve problems involving the volumes of right pyramids and right cones.



Volume is the amount of space an object occupies. It is measured in cubic units.

Capacity is the amount of material a container holds. It is measured in cubic units or capacity units.

Make Connections

Right pyramids and right cones are related to right prisms and right cylinders.
Look at the objects above.
How do you determine the volume of a right prism?
How do you determine the volume of a right cylinder?

Construct Understanding

TRY THIS

Work in a group.

You will need:

- a right cone and a right cylinder with equal bases and equal heights
- a right pyramid and a right prism with equal bases and equal heights
- a container of sand
- an empty container

A. Predict the relationship between the volume of the prism and the volume of the pyramid.

B. Fill the pyramid with sand; do not overfill. Pour the sand from the pyramid into the prism. How many full pyramids fill the prism?

C. What conclusion can you make about the relationship between the volumes of a right prism and a right pyramid with equal bases and equal heights?

D. Repeat Steps A to C for the cone and cylinder.

We use exponents when we write units for volume; for example, four cubic yards is 4 $yd.^3$ and four cubic metres is 4 m^3.

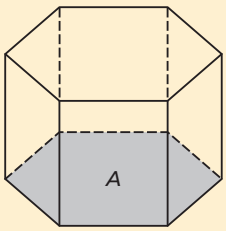
The volume of a right prism is 3 times the volume of a right pyramid with the same base and the same height.

Or, the volume of a right pyramid is $\frac{1}{3}$ the volume of the right prism with the same base and the same height.

Volumes of a Right Prism and a Right Pyramid

The volume of a right prism is:
Volume = (base area)(height)

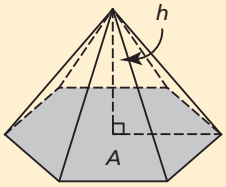
$V = Ah$



The volume of a right pyramid with the same base and the same height is:

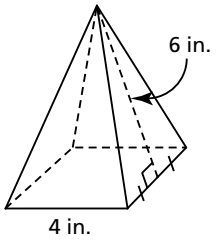
Volume $= \frac{1}{3}$(base area)(height)

$V = \frac{1}{3}Ah$



Example 1 Determining the Volume of a Right Square Pyramid Given Its Slant Height

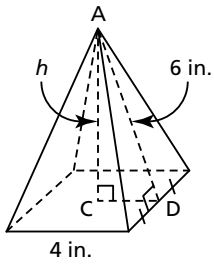
Calculate the volume of this right square pyramid to the nearest cubic inch.



SOLUTION

Calculate the height of the pyramid.
Let h inches represent the height.

In right $\triangle ACD$, CD is $\frac{1}{2}$ the side length of the base, so CD $= 2$ in.



Use the Pythagorean Theorem in right $\triangle ACD$ to calculate h.

$h^2 + 2^2 = 6^2$ Solve the equation for h^2.

$h^2 + 4 = 36$

$h^2 = 36 - 4$

$h^2 = 32$ Solve the equation for h.

$h = \sqrt{32}$

The height is $\sqrt{32}$ in.

Use the formula for the volume of a right pyramid:

Volume $= \frac{1}{3}$(base area)(height)

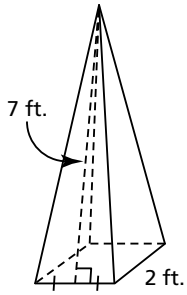
$V = \frac{1}{3}(4)^2(\sqrt{32})$

$V = 30.1698\ldots$

The volume of the pyramid is approximately 30 cubic inches.

CHECK YOUR UNDERSTANDING

1. Calculate the volume of this right square pyramid to the nearest cubic foot.



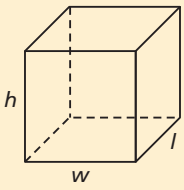
[Answer: approximately 9 ft.^{3}]

We can write algebraic formulas for the volumes of right rectangular prisms and right rectangular pyramids with the same base and the same height.

Volumes of a Right Rectangular Prism and a Right Rectangular Pyramid

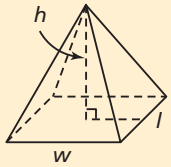
A right rectangular prism with length l, width w, and height h, has volume:

$V = lwh$



A right rectangular pyramid with base length l, base width w, and height h, has volume:

$V = \frac{1}{3}lwh$

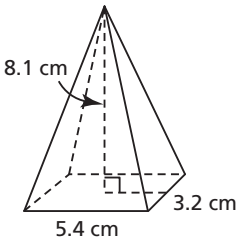


Example 2 Determining the Volume of a Right Rectangular Pyramid

Determine the volume of a right rectangular pyramid with base dimensions 5.4 cm by 3.2 cm and height 8.1 cm.
Answer to the nearest tenth of a cubic centimetre.

SOLUTION

Sketch and label a diagram.



Use the formula for the volume of right rectangular pyramid.

$V = \frac{1}{3}lwh$ Substitute: $l = 5.4$, $w = 3.2$, $h = 8.1$

$V = \frac{1}{3}(5.4)(3.2)(8.1)$

$V = 46.656$

The volume of the pyramid is approximately 46.7 cm^3.

CHECK YOUR UNDERSTANDING

2. Determine the volume of a right rectangular pyramid with base dimensions 3.6 m by 4.7 m and height 6.9 m. Answer to the nearest tenth of a cubic metre.

[Answer: approximately 38.9 m^3]

The volume of a right cylinder is 3 times the volume of a right cone with the same base and the same height.

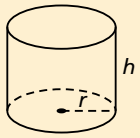
Or, the volume of a right cone is $\frac{1}{3}$ the volume of a right cylinder with the same base and the same height.

We can write algebraic formulas for the volumes of right cylinders and right cones with the same base and the same height.

Volumes of a Right Cylinder and a Right Cone

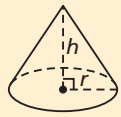
A right cylinder with base radius r and height h has volume:

$V = \pi r^2 h$



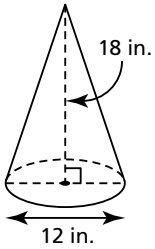
A right cone with base radius r and height h has volume:

$V = \frac{1}{3}\pi r^2 h$



Example 3 Determining the Volume of a Cone

Determine the volume of this cone to the nearest cubic inch.



SOLUTION

The radius, r, of the base of the cone is $\frac{1}{2}$ the diameter.

$r = \frac{1}{2}(12 \text{ in.})$

$r = 6$ in.

Use the formula for the volume of a cone.

$V = \frac{1}{3}\pi r^2 h$ Substitute: $r = 6$, $h = 18$

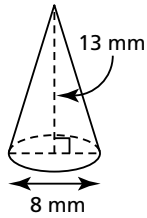
$V = \frac{1}{3}\pi(6)^2(18)$

$V = 678.5840\ldots$

The volume of the cone is approximately 679 cubic inches.

CHECK YOUR UNDERSTANDING

3. Determine the volume of this cone to the nearest cubic millimetre.



[Answer: approximately 218 mm^3]

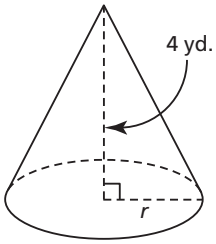
How can you determine if your answer is reasonable?

Example 4 Determining an Unknown Measurement

A cone has a height of 4 yd. and a volume of 205 cubic yards. Determine the radius of the base of the cone to the nearest yard.

SOLUTION

Sketch a diagram.



Use the formula for the volume of a cone.

$$V = \frac{1}{3}\pi r^2 h$$ Substitute: $V = 205$, $h = 4$

$$205 = \frac{1}{3}\pi r^2(4)$$ Solve for r^2. Multiply both sides by 3.

$$3(205) = 3\left(\frac{1}{3}\pi r^2(4)\right)$$

$$615 = 4\pi r^2$$ Divide both sides by 4π.

$$\frac{615}{4\pi} = \frac{4\pi r^2}{4\pi}$$

$$\frac{615}{4\pi} = r^2$$ Solve for r.

$$\sqrt{\frac{615}{4\pi}} = r$$

√(615/(4π))
6.995723336

$$r = 6.9957\ldots$$

The radius of the base of the cone is approximately 7 yd.

CHECK YOUR UNDERSTANDING

4. A cone has a height of 8 m and a volume of 300 m^3. Determine the radius of the base of the cone to the nearest metre.

[Answer: approximately 6 m]

Discuss the Ideas

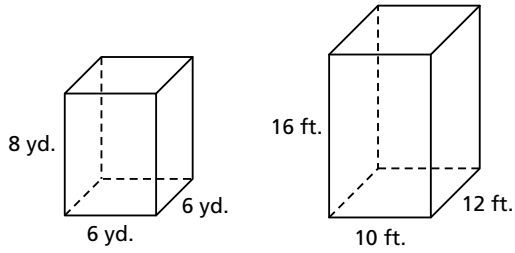
1. How are the volumes of a right pyramid and right cone related to the volumes of a right prism and right cylinder, respectively?
2. How are the formulas for the volumes of a right cone and a right pyramid alike? How are the formulas different?
3. Suppose you cannot remember the formula for the volume of a right cone or a right pyramid. What strategy could you use to determine the volume?

Exercises

A

4. Calculate the volume of each right prism.

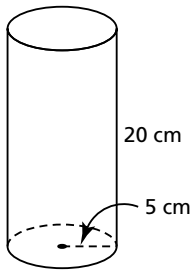
a) square prism **b)** rectangular prism



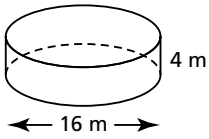
5. For each prism in question 4, sketch a right pyramid with the same base and same height. Calculate the volume of each pyramid.

6. Calculate the volume of each right cylinder to the nearest cubic unit.

a)



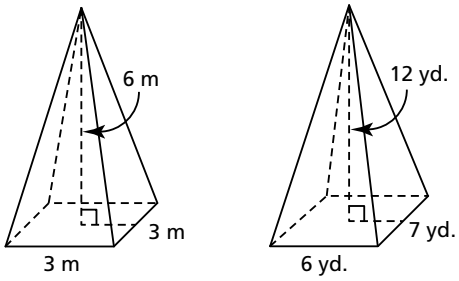
b)



7. For each cylinder in question 6, sketch a right cone with the same base and same height. Calculate the volume of each cone to the nearest cubic unit.

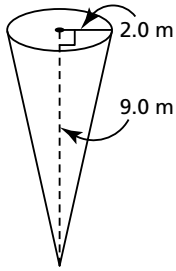
8. Calculate the volume of each right pyramid.

a) square pyramid **b)** rectangular pyramid

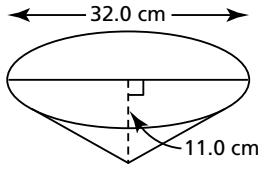


9. Calculate the volume of each right cone. Write the answer to the nearest tenth of a cubic unit.

a)



b)



B

10. A regular tetrahedron has base area 68.0 m^2 and height 10.2 m.

a) Sketch the tetrahedron.

b) Determine its volume to the nearest tenth of a cubic metre.

11. A right cone has slant height 12 yd. and base diameter 4 yd.

a) Sketch the cone.

b) Determine its volume to the nearest cubic yard.

12. A stone monument has the shape of a square pyramid. Its slant height is 1.6 m and the side length of its base is 0.8 m. Determine the volume of the monument to the nearest tenth of a cubic metre.

13. Annika has a wooden right rectangular pyramid. She measures the dimensions of the base as 10.4 cm by 8.6 cm, and the height as 14.8 cm.

a) Explain how Annika can use these measurements to calculate the volume of the pyramid.

b) What is the volume of the pyramid to the nearest tenth of a cubic centimetre?

14. An ice cream shop in Bellevue, Alberta, created a new dessert. It is a waffle cone with a height of 5 in. and a base diameter of 2 in., filled with ice cream. Then whipped topping and sprinkles are added.

a) The ice cream is level with the top of the cone. How much ice cream can the cone hold? Write the answer to the nearest cubic inch.

b) One cubic inch of soft ice cream costs 55¢, the waffle cone costs 35¢, and the whipped topping and sprinkles cost 10¢ per dessert. How much will this dessert cost to produce?

c) Suppose the cone had the shape of a right square pyramid with base side length 2 in. and height 5 in. How much ice cream would it hold?

15. A right square pyramid has a base side length of 3.5 m. Each triangular face has two equal sides of length 4.5 m.

a) Sketch and label the pyramid.

b) Calculate the height of the pyramid to the nearest tenth of a metre.

c) Calculate the volume of the pyramid to the nearest tenth of a cubic metre.

16. Determine the volume of a right rectangular pyramid with base dimensions 6 ft. by 12 ft. For each triangular face, the equal sides have length 6 yd. Write the answer to the nearest cubic foot.

17. A tea bag has the shape of a regular tetrahedron. Each edge is 5.8 cm long. The height of the tetrahedron is approximately 4.7 cm.

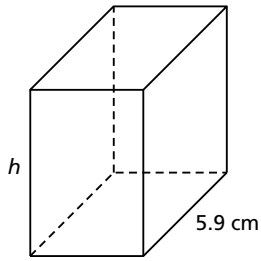
a) Calculate the area of the base of the tetrahedron to the nearest square centimetre.

b) What is the volume of the tea bag to the nearest cubic centimetre?

c) Do you think the volume of tea in the bag is equal to your answer from part b? Explain.

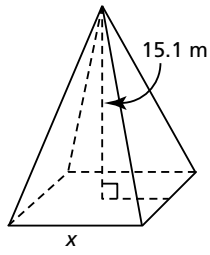
18. For each object, its volume, V, and some dimensions are given. Calculate the dimension indicated by the variable. Write each answer to the nearest tenth of a unit.

a) right rectangular prism



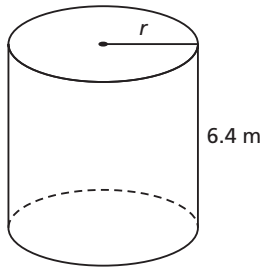
$V = 88.8 \text{ cm}^3$

b) right square pyramid



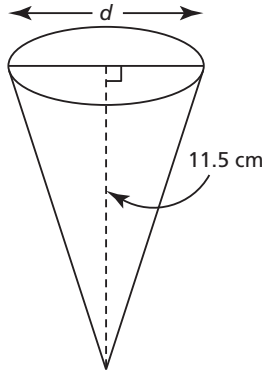
$V = 554.9 \text{ m}^3$

c) right cylinder



$V = 219.0 \text{ m}^3$

d) right cone



$V = 164.9 \text{ cm}^3$

19. Sunil immersed a right plastic cone in a measuring cylinder containing water and determined that the volume of the cone was 33.5 cm^3. He measured the diameter of the base of the cone as 4.0 cm.

a) With these data, how could Sunil calculate the height of the cone?

b) What is the height of the cone to the nearest tenth of a centimetre?

C

20. An underground tank has the shape of a right cone, supported with its apex beneath its base. The tank collects the water run-off for a three-storey parking garage. The cone has a base diameter of 5.0 m and a height of 3.5 m. ($1\ m^3 = 1\ kL$)

a) What is the capacity of this tank to the nearest tenth of a kilolitre?

b) How much water is in the tank when the water level is 1 m below the top of the tank?

21. A right square pyramid has a volume of 111 cubic yards. The base has a side length of 6 yd. Determine the slant height of the pyramid to the nearest yard.

22. A right rectangular pyramid has base dimensions 5 m by 3 m, and a height of 10 m. A horizontal cut is made through the pyramid 2 m from its apex and this smaller right rectangular pyramid is removed. What is the volume of the remaining piece?

Reflect

How are a cone and a pyramid alike? How are they different? Explain how these similarities and differences account for the formulas for the volume of each object.

THE WORLD OF MATH

Careers: Petroleum Engineer

Petroleum engineers work closely with geologists, geophysicists, and oilfield operating personnel. Their primary job is to analyze drilling data to determine if wells contain significant quantities of oil and gas. The engineers estimate the costs of developing a drilling location and its potential profits. After a site has been selected for drilling, a petroleum engineer designs and implements appropriate drilling, processing, and transportation for the oil or gas. To do this, petroleum engineers must be able to measure lengths and distances accurately, estimate the recoverable volumes of oil and gas based on seismic data, and determine the most efficient methods for storage and transportation to minimize any environmental impact.

1.6 Surface Area and Volume of a Sphere

LESSON FOCUS

Solve problems involving the surface area and volume of a sphere.

Make Connections

The sphere above is in Winnipeg, Manitoba. It contains methane and carbon dioxide that is produced during the treatment of waste water. How could you estimate the volume of gas the sphere could hold?

Construct Understanding

TRY THIS

Work with a partner.

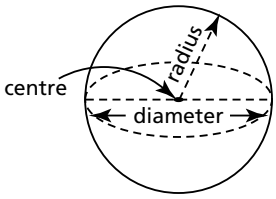
You will need:

- an orange
- calipers or a ruler
- a compass

A. An orange approximates a sphere and the area of its peel represents the surface area of the sphere.
Measure and record the diameter of the orange in 3 different places.
Calculate the mean diameter of the orange.

B. Draw 6 circles with diameter equal to the mean diameter of the orange.

A **sphere** is the set of points in space that are the same distance from a fixed point, which is the *centre*. A line segment that joins the centre to any point on the sphere is a *radius*. A line segment that joins two points on a sphere and passes through the centre is a *diameter*.



C. Peel the orange and arrange the peel within the circles. Completely fill one circle before moving to the next one. (Hint: It is easiest to use small pieces of peel).

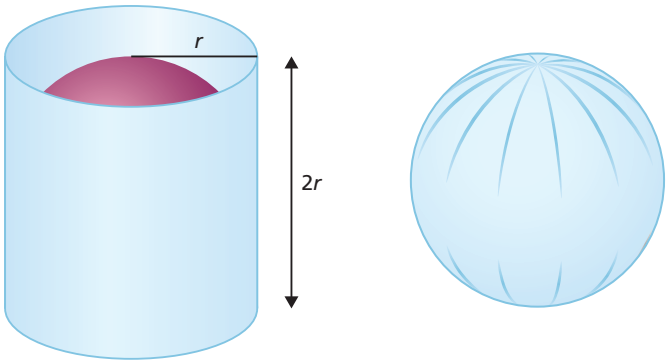
D. Continue filling the circles until all the peel has been used. About how many circles did you cover with orange peel?

E. Use the filled circles and the formula for the area of a circle to estimate the surface area of the orange.

F. Compare your answer for part D with those of your classmates. Work together to create a formula for determining the surface area of a sphere.

The surface area of a sphere is related to the curved surface area of a cylinder that encloses it. The cylinder has the same diameter as the sphere, and a height equal to its diameter.

If the curved surface of the cylinder is made from paper, it can be cut and pasted on the surface of the sphere to cover it.



The curved surface area, SA_C, of a cylinder with base radius r and height h is:
$SA_C = 2\pi rh$

When a cylinder has base radius r and height $2r$:

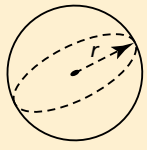
$SA_C = 2\pi r(2r)$

$SA_C = 4\pi r^2$

So, this is also the formula for the surface area of a sphere with radius r.

Surface Area of a Sphere

The surface area, SA, of a sphere with radius r is:
$SA = 4\pi r^2$



Example 1 Determining the Surface Area of a Sphere

The diameter of a baseball is approximately 3 in. Determine the surface area of a baseball to the nearest square inch.

SOLUTION

Use the formula for the surface area of a sphere.

The radius is:

$\frac{1}{2}(3 \text{ in.}) = 1.5 \text{ in.}$

$SA = 4\pi r^2$ Substitute $r = 1.5$.

$SA = 4\pi(1.5)^2$

$SA = 28.2743\ldots$

The surface area of a baseball is approximately 28 square inches.

CHECK YOUR UNDERSTANDING

1. The diameter of a softball is approximately 4 in. Determine the surface area of a softball to the nearest square inch.

[Answer: approximately 50 in.^{2}]

What happens to the surface area of a sphere when its radius is doubled?

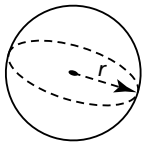
Example 2 Determining the Diameter of a Sphere

The surface area of a lacrosse ball is approximately 20 square inches. What is the diameter of the lacrosse ball to the nearest tenth of an inch?

SOLUTION

Let r represent the radius of the lacrosse ball.

Use the formula for the surface area of a sphere.



$SA = 4\pi r^2$ Substitute $SA = 20$.

$20 = 4\pi r^2$ Divide both sides of the equation by 4π.

$\frac{20}{4\pi} = \frac{4\pi r^2}{4\pi}$

$\frac{20}{4\pi} = r^2$ Solve for r.

$r = \sqrt{\frac{20}{4\pi}}$

$r = 1.2615\ldots$

(Solution continues.)

CHECK YOUR UNDERSTANDING

2. The surface area of a soccer ball is approximately 250 square inches. What is the diameter of a soccer ball to the nearest tenth of an inch?

[Answer: approximately $8\frac{9}{10}$ in.]

The diameter is:

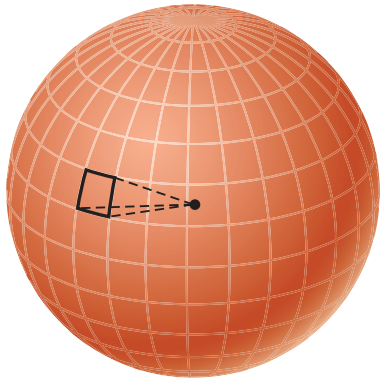
$2r = 2(1.2615\ldots)$

$2r = 2.5231\ldots$

The diameter of the lacrosse ball is approximately $2\frac{5}{10}$ in., or $2\frac{1}{2}$ in.

We can use the formula for the surface area of a sphere to develop a formula for the volume of a sphere.

Visualize a sphere covered with very small congruent squares, and each square is joined by line segments to the centre of the sphere to form a square pyramid. The volume of the sphere is the sum of the volumes of the square pyramids.



Volume of sphere = sum of volumes of pyramids

Volume of sphere = sum of all the $\left[\frac{1}{3}(\text{base area})(\text{height})\right]$

The height of a pyramid is the radius of the sphere.

Volume of sphere $= \frac{1}{3}(\text{sum of all the base areas})(r)$

The sum of all the base areas is the surface area of the sphere: $4\pi r^2$

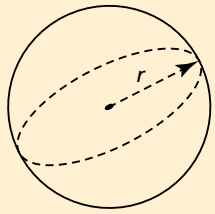
$V = \frac{1}{3}(4\pi r^2)(r)$

$V = \frac{4}{3}\pi r^3$

Volume of a Sphere

The volume, V, of a sphere with radius r is:

$V = \frac{4}{3}\pi r^3$



Example 3 Determining the Volume of a Sphere

The sun approximates a sphere with diameter 870 000 mi. What is the approximate volume of the sun?

SOLUTION

Use the formula for the volume of a sphere.

The radius, r, is:

$r = \frac{1}{2}(870\,000 \text{ mi.})$

$r = 435\,000 \text{ mi.}$

$V = \frac{4}{3}\pi r^3$ Substitute $r = 435\,000$.

$V = \frac{4}{3}\pi (435\,000)^3$

$4/3 \times \pi \times 435000\text{^}3$
$3.447913645 \times 10^{17}$

$V = 3.4479\ldots \times 10^{17}$ This number is too large for the calculator display, so it is displayed as the product of a number and a power of 10.

The volume of the sun is approximately 3.4×10^{17} cubic miles.

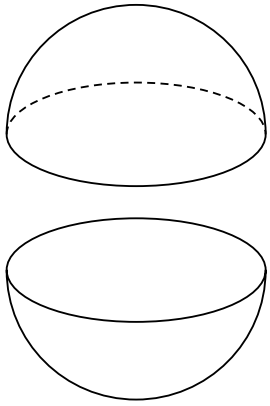
CHECK YOUR UNDERSTANDING

3. The moon approximates a sphere with diameter 2160 mi. What is the approximate volume of the moon?

[Answer: approximately 5.3×10^9 mi.^{3}]

When a large number is displayed as the product of a power of 10 and a number between 1 and 10, the number is written in *scientific notation*.

When a sphere is cut in half, two *hemispheres* are formed.



hemispheres

Why is a globe constructed from two hemispheres?

Example 4 Determining the Surface Area and Volume of a Hemisphere

A hemisphere has radius 8.0 cm.

a) What is the surface area of the hemisphere to the nearest tenth of a square centimetre?

b) What is the volume of the hemisphere to the nearest tenth of a cubic centimetre?

SOLUTION

a) SA of a hemisphere = SA of one-half a sphere + area of a circle

$SA = \frac{1}{2}(4\pi r^2) + \pi r^2$

$SA = 2\pi r^2 + \pi r^2$

$SA = 3\pi r^2$ Substitute: $r = 8.0$

$SA = 3\pi(8.0)^2$

$SA = 603.1857\ldots$

The surface area of the hemisphere is approximately 603.2 cm^2.

b) Volume of a hemisphere = volume of one-half a sphere

$V = \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$V = \frac{2}{3}\pi r^3$ Substitute: $r = 8.0$

$V = \frac{2}{3}\pi(8.0)^3$

$V = 1072.3302\ldots$

The volume of the hemisphere is approximately 1072.3 cm^3.

CHECK YOUR UNDERSTANDING

4. A hemisphere has radius 5.0 cm.

a) What is the surface area of the hemisphere to the nearest tenth of a square centimetre?

b) What is the volume of the hemisphere to the nearest tenth of a cubic centimetre?

[Answers: a) approximately 235.6 cm^2 b) approximately 261.8 cm^3]

In *Example 4a*, why can we write $2\pi r^2 + \pi r^2$ as $3\pi r^2$?

Discuss the Ideas

1. A sphere is cut in half. How is the surface area of the sphere related to the area of the circular face on one hemisphere?

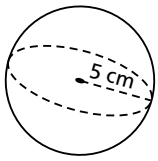
2. Visualize a sphere with radius r that fits in a cylinder with base radius r and height $2r$.
How is the volume of the sphere related to the volume of the cylinder?
How could you use this relationship to help you remember the formula for the volume of a sphere?

Exercises

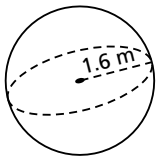
A

3. Determine the surface area of each sphere to the nearest square unit.

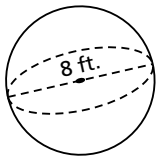
a)



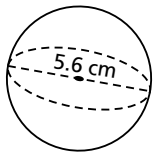
b)



c)



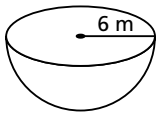
d)



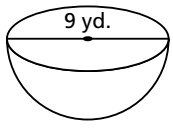
4. Determine the volume of each sphere in question 3 to the nearest cubic unit.

5. Determine the surface area and volume of each hemisphere. Write your answers to the nearest whole unit.

a)



b)



B

6. Use a marble or other sphere. Measure its diameter. Calculate its volume and surface area.

7. A sphere has a radius of 8.4 m. Determine its surface area and volume to the nearest tenth of a unit.

8. The surface area of a tennis ball is approximately 127 cm^2. What is the radius of the tennis ball to the nearest tenth of a centimetre?

9. A sphere has a surface area of 452 square inches. What is the diameter of the sphere to the nearest inch?

10. A glass bowl approximates a hemisphere with diameter 20 cm.

a) What is the capacity of the bowl to the nearest tenth of a litre? (1000 cm^3 = 1 L)

b) One cup is 250 mL. How many cups of punch can the bowl hold?

11. A sphere has a diameter of 12 cm. A hemisphere has a radius of 8 cm.

a) Which object has the greater surface area?

b) Which object has the greater volume?

12. The gas storage sphere on page 45 has diameter 15.8 m.

a) What is the surface area of the sphere to the nearest square metre?

b) What is the capacity of the sphere to the nearest kilolitre? (1 kL = 1 m^3)

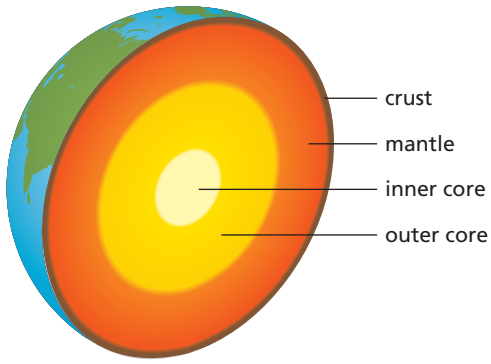
13. Earth approximates a sphere but its diameter varies. The mean diameter of Earth is approximately 12 756 km.

a) Determine the surface area of Earth to the nearest square kilometre.

b) About 70% of Earth's surface is covered in water. What is this area in square kilometres?

c) Determine the volume of Earth to the nearest thousand cubic kilometres.

d) The inner core of Earth has a radius of approximately 1278 km. Determine, to the nearest thousand cubic kilometres, the volume of Earth that is *not* part of the inner core.



14. The diameter of Earth through the North and South poles is 16 km less than its mean diameter, approximately 12 756 km. The diameter of Earth at the equator is 26 km greater than its mean diameter. Determine the approximate volume of Earth using the polar radius and equatorial radius.

15. The centre of a doughnut is removed and formed to make a sphere of dough with diameter 2.5 cm. A batch of these spheres is to be covered in a sugar glaze. There is enough glaze to cover an area of 4710 cm^2. How many spheres can be glazed?

16. The size of a ball used in sport is often described by the measure of its circumference. The circumference of a ball is the length of the longest circle that can be drawn on the surface of the ball. A volleyball has a circumference of 66 cm and a basketball has a circumference of $29\frac{1}{2}$ in.

a) Determine the radius of each ball to the nearest unit.
b) Determine the surface area of each ball to the nearest square unit.
c) Determine the volume of each ball to the nearest cubic unit.
d) Which ball is larger? Justify your answer.

17. In the rain forests of Vancouver Island, there are tree houses shaped like spheres. One spherical shell has outside diameter 3.20 m and inside diameter 3.15 m.

a) Calculate the volume of the inside of the shell to the nearest tenth of a cubic metre.
b) What is the difference between the outside and inside surface areas, to the tenth of a square metre?

18. A hemisphere has a circumference of 47.1 m. Determine the surface area and volume of the hemisphere to the nearest tenth of a unit.

19. A fitness ball is delivered in a flat package with a hand pump. The pump inflates the ball at a rate of 280 cm^3 per pump, to a diameter of 28 cm. How many pumps are needed to inflate the ball? Justify your answer.

20. A pail of cookie dough is cylindrical, with diameter 17 cm and height 13 cm. A scoop makes a sphere of cookie dough with diameter 5 cm. How many cookies can be made from this pail of dough?

C

21. Giselle has a block of wood that measures 14 cm by 12 cm by 10 cm. She is making a wooden ball in tech class.

a) What percent of the wood is wasted?
b) What assumptions are you making?

22. Derive formulas for the surface area and volume of a sphere in terms of its diameter d. Check your formulas by using them to calculate the surface area and volume of the sphere in question 3c.

23. A beach ball that was deflated to 70% of its maximum volume now has a volume of 420 cubic inches. What is the radius of the beach ball when it is at its maximum volume?

24. A spherical balloon has a radius of 10 cm. It is blown up until its radius is three times the original radius. For the inflated balloon and the original balloon:

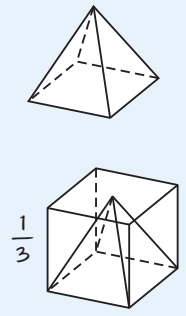
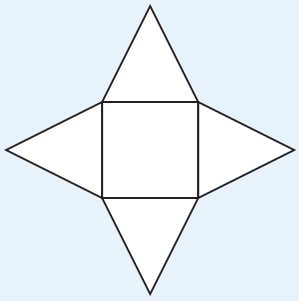
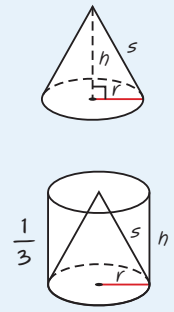
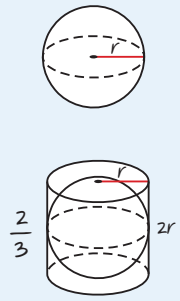
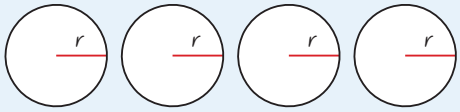
a) How do the circumferences compare?
b) How do the surface areas compare?
c) How do the volumes compare?

Reflect

What strategies do you have for remembering the formulas for the volume and surface area of a sphere?

CHECKPOINT 2

Connections

Volume	Surface Area
Right pyramid with regular polygon base  	 $SA = \frac{1}{2}$(slant height)(perimeter of base) + base area
Right cone  	SA = lateral area + base area $SA = \pi rs + \pi r^2$
Sphere  	

Concept Development

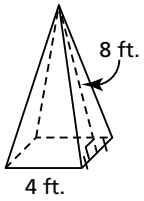
- **In Lesson 1.4**
  - You developed and applied formulas for the **surface areas of a right pyramid and a right cone**.
  - You **determined an unknown dimension** given the surface area of a right cone or a right pyramid, and sufficient other information.
- **In Lesson 1.5**
  - You described the **volume relationships between a right prism and a right pyramid, and between a right cylinder and a right cone.**
  - You developed and applied formulas for the **volumes of a right pyramid and a right cone.**
  - You **determined an unknown dimension** given the volume of an object and sufficient other information.
- **In Lesson 1.6**
  - You developed and applied formulas for the **surface area and volume of a sphere**.
  - You **determined an unknown dimension** given the surface area of a sphere and sufficient other information.

Assess Your Understanding

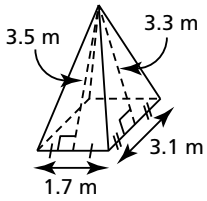
1.4

1. Determine the surface area of each object to the nearest square unit.

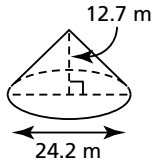
a) right square pyramid



b) right rectangular pyramid



c) right cone



2. The top of a clock tower has the shape of a right square pyramid, with base side length 15 m and height 12 m. The roofing tiles on the tower need to be replaced. Determine the area that needs to be re-covered, to the nearest square metre.

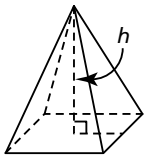
3. A sector of a circle is used to make a hat shaped like a right cone. The cone has a height of 14 in. and a base diameter of 8 in. The outside of the hat is to be covered in red crepe paper. Determine the area that will be covered in crepe paper to the nearest square inch.

1.5

4. Determine the volume of each object in question 1 to the nearest cubic unit.

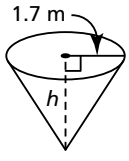
5. For each object, its volume, V, and some dimensions are given. Calculate the dimension indicated by the variable to the nearest tenth of a unit.

a) right square pyramid



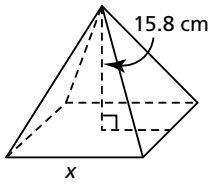
$V = 168.8 \text{ cm}^3$

b) right cone



$V = 8.2 \text{ m}^3$

c) right square pyramid



$V = 1594.5 \text{ cm}^3$

1.6

6. Determine the surface area and volume of each object with the given dimension. Write the answers to the nearest tenth of a unit.

a) sphere, radius 8.8 km

b) hemisphere, diameter 6.8 cm

7. A spherical globe has circumference 158 cm. The surface of the globe is to be painted with a high-gloss varnish. What is the area to be painted to the nearest square centimetre?

1.7 Solving Problems Involving Objects

LESSON FOCUS

Solve problems involving the surface area and volume of composite objects.

Make Connections

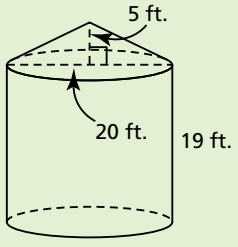
A farmer is constructing a new grain bin. The bin she would like to build has a cylindrical body and a cone-shaped roof. The farmer knows the dimensions of the bin she wants to build.

How can the farmer determine the amount of material she will need to build the bin?

Construct Understanding

THINK ABOUT IT

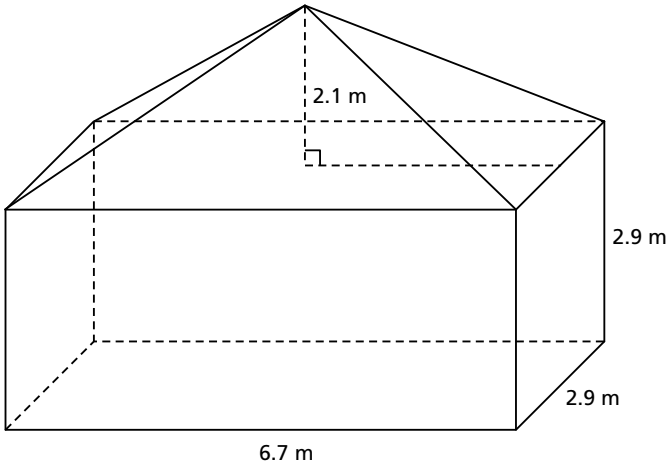
Here is a sketch of a grain bin. The farmer's grain truck can hold 550 cubic feet of barley. How many truckloads are required to fill the bin?



A *composite object* comprises two or more distinct objects.
To determine the volume of a composite object, identify the distinct objects, calculate the volume of each object, then add the volumes.

Example 1 Determining the Volume of a Composite Object

Determine the volume of this composite object to the nearest tenth of a cubic metre.



SOLUTION

The object comprises a right rectangular prism and a right rectangular pyramid.

Use the formula for the volume of a right rectangular prism.

$V = lwh$ Substitute: $l = 6.7$, $w = 2.9$, $h = 2.9$

$V = (6.7)(2.9)(2.9)$

$V = 56.347$

Use the formula for the volume of a right rectangular pyramid.

$V = \frac{1}{3}lwh$ Substitute: $l = 6.7$, $w = 2.9$, $h = 2.1$

$V = \frac{1}{3}(6.7)(2.9)(2.1)$

$V = 13.601$

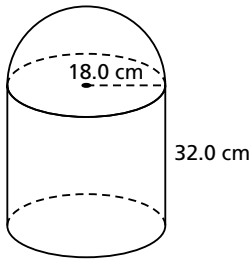
Volume of the composite object is:

$56.347 + 13.601 = 69.948$

The volume of the composite object is approximately 69.9 m^3.

CHECK YOUR UNDERSTANDING

1. Determine the volume of this composite object to the nearest tenth of a cubic centimetre.

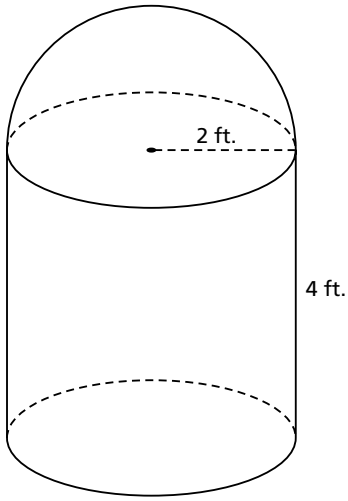


[Answer: approximately 44 786.5 cm^3]

To calculate the surface area of a composite object, the first step is to determine the faces that comprise the surface area. Then calculate the sum of the areas of these faces.

Example 2 Determining the Surface Area of a Composite Object

Determine the surface area of this composite object to the nearest square foot.



SOLUTION

The curved surface of the hemisphere, one base of the cylinder, and the curved surface of the cylinder comprise the surface area.

The cylinder and hemisphere have equal radii.

Surface area of composite object is:
Area of curved surface of hemisphere + area of base of cylinder + area of curved surface of cylinder

Use the algebraic formulas for surface area.

$SA = 2\pi r^2 + \pi r^2 + 2\pi rh$

$SA = 3\pi r^2 + 2\pi rh$ Substitute: $r = 2$, $h = 4$

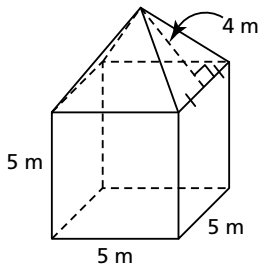
$SA = 3\pi(2)^2 + 2\pi(2)(4)$

$SA = 87.9645\ldots$

The surface area of the composite object is approximately 88 square feet.

CHECK YOUR UNDERSTANDING

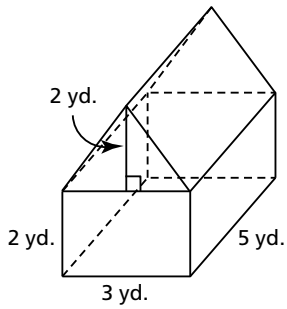
2. Determine the surface area of this composite object.



[Answer: 165 m^2]

Example 3 Solving a Problem Related to a Composite Object

A cabane à sucre is a composite object formed by a rectangular prism with a right triangular prism as its roof. Determine the surface area of the cabane à sucre in square yards.

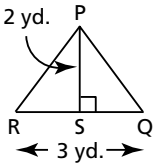


SOLUTION

The surface area of the cabane is the sum of the areas of the 4 walls, plus the areas of the 2 rectangular faces and 2 triangular faces of the roof.

The area of the 4 walls, in square yards, is:
$A = 2(3)(2) + 2(5)(2)$
$A = 32$

To determine the surface area of the roof, calculate the width of a rectangular face.
Sketch a triangle to represent a triangular face of the roof.



Use the Pythagorean Theorem in right △PSQ.

$PQ^2 = PS^2 + SQ^2$ Substitute the known values.
$PQ^2 = 2^2 + 1.5^2$
$PQ^2 = 6.25$
$PQ = \sqrt{6.25}$
$PQ = 2.5$

Area of the 2 rectangular faces of the roof, in square yards, is:
$A = 2(2.5)(5)$
$A = 25$

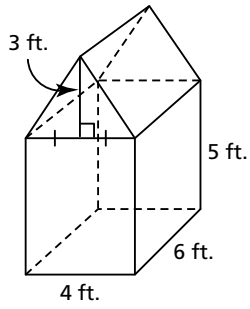
Area of the 2 triangular faces of the roof, in square yards, is:
$A = 2\left(\frac{1}{2}\right)(3)(2)$
$A = 6$

Surface area of the cabane à sucre, in square yards, is:
$SA = 32 + 25 + 6 = 63$

The surface area of the cabane à sucre is 63 square yards.

CHECK YOUR UNDERSTANDING

3. A tool shed is formed by a rectangular prism with a triangular prism as its roof. Determine the surface area of the tool shed to the nearest square foot.



[Answer: approximately 155 ft.^{2}]

Discuss the Ideas

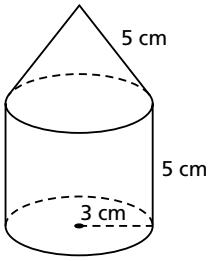
1. When you determine the surface area of a composite object, how do you identify the faces that comprise the surface area?
2. When might you use the Pythagorean Theorem in a calculation of the surface area or volume of a composite object? How do you know your answer is reasonable?

Exercises

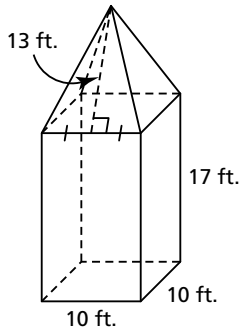
A

3. Determine the surface area of each composite object to the nearest square unit.

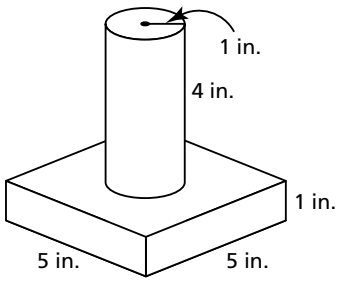
a) right cylinder and right cone



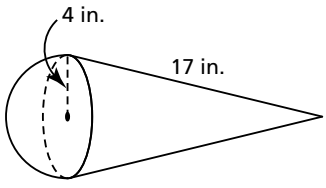
b) right square prism and right square pyramid



c) right square prism and right cylinder



d) right cone and hemisphere



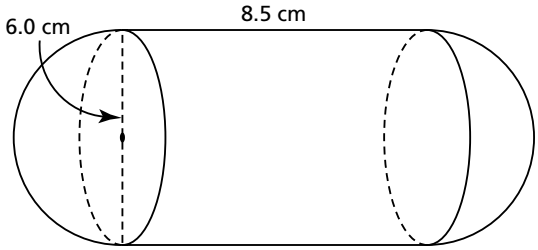
4. **a)** For which composite objects in question 3 could you calculate the volumes without determining any further dimensions?

b) Determine the volume of each composite object you identified in part a.

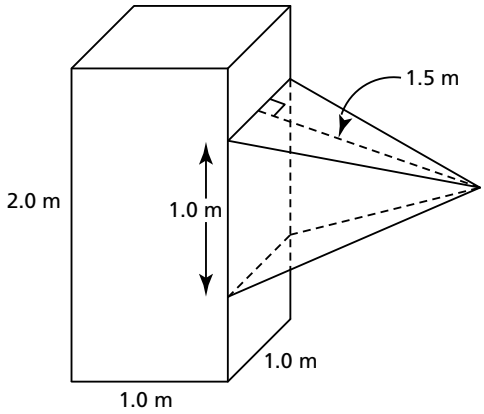
B

5. Determine the surface area and volume of each composite object. Write the answers to the nearest tenth of a unit.

a) right cylinder and hemispheres

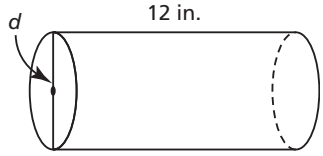


b) right square prism and right square pyramid



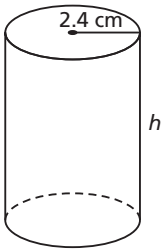
6. For each object, its surface area, *SA*, and some dimensions are given. Calculate the dimension indicated by the variable. Write each answer to the nearest tenth of a unit.

a) right cylinder



curved $SA = 219$ in.^{2}

b) right cylinder



total $SA = 137.2$ cm^2

7. A rocket has a cylindrical body and a cone-shaped nose. The cylinder is 55 cm long with a radius of 6 cm. The cone has a slant height of 12 cm and has the same radius as the cylinder.

a) Sketch and label a diagram of the rocket.

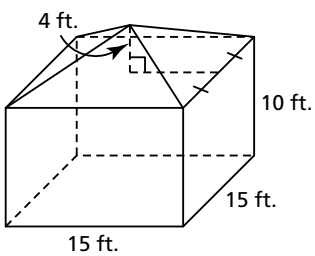
b) Determine the surface area of the rocket to the nearest square centimetre.

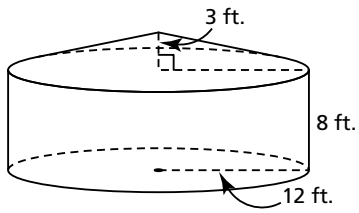
c) Determine the volume of the rocket to the nearest cubic centimetre.

d) One-third of the interior space of the rocket is used for fuel storage. How much fuel can the rocket hold?

8. A solid sphere just fits inside a cube that has an edge length equal to the diameter of the sphere. The edge length of the cube is 5.8 cm. What is the volume of air in the cube to the nearest cubic centimetre?

9. Here are two different grain storage bins.



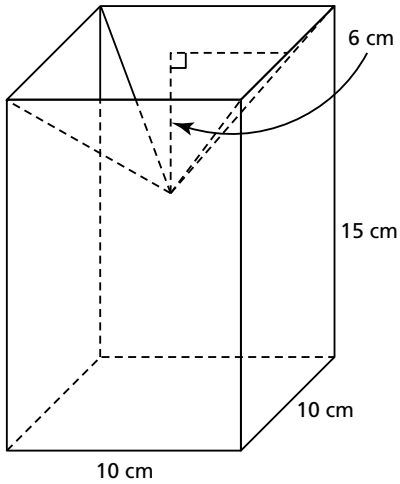


a) Which storage bin holds more grain?

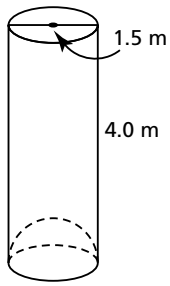
b) Each storage bin has a cement base. The materials for the walls and roof of the square-based bin cost \$10.49 per square foot. The materials for the walls and roof of the circular-based bin cost \$9.25 per square foot. Which bin is cheaper to build? Justify your answer.

10. Determine the volume of each object to the nearest tenth of a cubic unit.

a) a right square prism with a right square pyramid removed



b) a right cylinder with a hemisphere removed



11. Determine the surface area of each object in question 10 to the nearest tenth of a square unit.

C

12. An igloo approximates a hemisphere, with an entrance tunnel that approximates half a right cylinder.

The base of the igloo has diameter 4.0 m. The entrance is 0.8 m long; it has an external radius of 0.8 m and an internal radius of 0.7 m. Calculate the surface area of the outside of the igloo and tunnel.

13. An ice sculpture can be made by pouring water into a mould or by carving blocks of ice.

a) One mould forms a sculpture that is a composite object comprising a right cylinder with base diameter 15 in. and height 3 in., and a right cone with the same base diameter as the base of the cylinder and a height of 9 in. Determine the volume of the sculpture to the nearest cubic inch.

b) The sculpture in part a is carved out of a block of ice with the shape of a right square prism. What are the least possible dimensions for the prism to the nearest inch?

c) The sculpture in part a is carved from a block of ice with the shape of a right rectangular prism with dimensions 16 in. by 15 in. by 12 in. What volume of ice, in cubic inches, remains?

Reflect

Which do you find easier to calculate: the surface area of a composite object or its volume? Explain your choice.

THE WORLD OF MATH

Profile: Festival du Voyageur

The Festival du Voyageur is an annual event that takes place in Winnipeg every February to celebrate the city's Francophone and Métis cultural heritage. Major attractions at the festival are the snow sculptures that are displayed at Voyageur Park and in neighbourhoods around the city. The festival also includes an International Snow Sculpting Symposium, where teams of sculptors create unique artwork from blocks of snow measuring 3.0 m by 3.7 m by 3.7 m. Each year, sculptors transform 450 000 cubic feet of snow into a winter wonderland.

What is the volume of snow in a sculpture that measures 50 ft. by 18 ft. by 6 ft.?

STUDY GUIDE

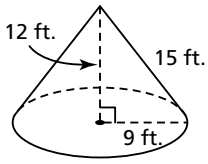
CONCEPT SUMMARY

Big Ideas	Applying the Big Ideas
■ You can use proportional reasoning to convert measurements.	■ Converting within and between two systems of measurement allows you to measure lengths using the most appropriate unit from either system.
■ The volume of a right pyramid or cone is related to the volume of the enclosing right prism or cylinder.	■ The volume of a right pyramid or a right cone is $\frac{1}{3}$ the volume of its enclosing right prism or right cylinder.
■ The surface area of a right pyramid or cone is the sum of the areas of the faces and the curved surfaces.	■ You can determine the surface area of a right pyramid by sketching a labelled diagram of its net and then calculating the area of each triangle and polygon that forms the net. For a right pyramid with a regular polygon base, the triangular faces are congruent. ■ For a right pyramid with a regular polygon base and for a right cone, the surface area is: $\frac{1}{2}$ (slant height)(perimeter of base) + (base area)
■ The surface area of a sphere is related to the curved surface area of the enclosing cylinder.	■ The surface area of a sphere is equal to the curved surface area of the cylinder that encloses it.

Reflect on the Chapter

- Why do we need a standard system of measurement?
- What strategies do you have for using a referent to estimate a length?
- What is surface area? Describe how to determine the surface areas of three different shaped objects.
- What is volume? Choose two objects. Explain how their volumes are related.

SKILLS SUMMARY

Skill	Description	Example
Convert within and between systems of measurement. [1.1, 1.3]	Use proportional reasoning to convert. Use unit analysis to check the conversion.	Convert 2 mi. to feet. Use the table on page 6. There are 5280 ft. in 1 mi. So, 2 mi. = 2(5280 ft.) = 10 560 ft.
Choose an appropriate measuring instrument and unit to measure an object. [1.2]	The measuring device could be a ruler, calipers, or tape measure. The units could be imperial or metric.	Which measuring device is most appropriate for measuring the length of your index finger? Which unit would you use? I would use a ruler and measure in centimetres.
Calculate the surface area of a right prism, right pyramid, right cylinder, right cone, sphere, or a composite object. [1.4, 1.6, 1.7]	To determine surface area: **1.** Identify the faces that comprise the surface area. **2.** Calculate the area of each face. **3.** Add the areas. You may need to use the Pythagorean Theorem to determine any dimension that is not given.	Determine the surface area of this cone.  $SA = \pi r^2 + \pi rs$ $SA = \pi(9)^2 + \pi(9)(15)$ $SA = 678.5840\ldots$ Surface area is approximately 679 square feet.
Calculate the volume of a right prism, right pyramid, right cylinder, right cone, sphere, or a composite object. [1.5, 1.6, 1.7]	To determine volume: **1.** Identify the objects. **2.** Use a formula to determine the volume of each object. **3.** Add the volumes, if necessary. You may need to use the Pythagorean Theorem to determine any dimension that is not given.	Determine the volume of the cone above. $V = \frac{1}{3}\pi r^2 h$ $V = \frac{1}{3}\pi(9)^2(12)$ $V = 1017.8760\ldots$ The volume is approximately 1018 cubic feet.

REVIEW

1.1

1. Which imperial unit is the most appropriate unit to measure each item? Justify your choice.
a) the length of your arm
b) the width of the classroom
c) the distance you ran in gym class

2. For each item in question 1a and b:
a) Use a referent to estimate its measure.
b) Use a ruler or tape measure to check your estimate.

3. Convert:
a) 14 yd. to feet
b) 5 mi. to yards
c) 6 ft. 3 in. to inches
d) 123 in. to yards, feet, and inches

4. The scale of a model airplane is 1 in. to 40 in. The model is 8 in. long. How long is the actual plane?

1.2

5. Describe a strategy you would use to estimate then measure each length in imperial units and in metric units.
a) the diameter of a car tire
b) the length of a car
c) the radius of a marble

1.3

6. Convert each measurement:
a) 261 cm to feet and the nearest inch
b) 125 m to yards, feet, and the nearest inch
c) 6 km to miles and the nearest yard
d) 350 mm to feet and the nearest inch

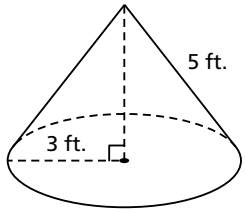
7. Convert each measurement. Answer to the nearest tenth.
a) 13 yd. 2 ft. to metres
b) 4 mi. 350 yd. to kilometres
c) 1 ft. 7 in. to centimetres
d) $8\frac{1}{2}$ in. to millimetres

8. The length of Vancouver Island from the north to the south is approximately 460 km. Sarah has an average stride length of 27 in. How many strides would Sarah take to walk from the northernmost tip to the southernmost tip?

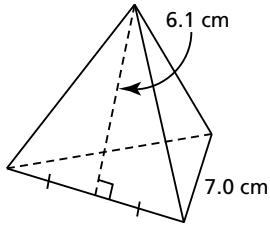
1.4

9. Determine the surface area of each object to the nearest square unit.

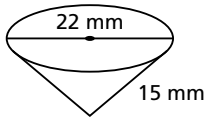
a) right cone



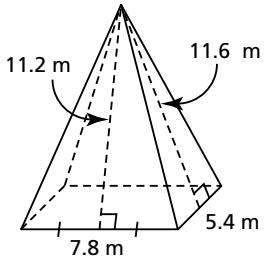
b) regular tetrahedron



c) right cone



d) right rectangular pyramid



10. A right rectangular pyramid has base dimensions 7 yd. by 5 yd. and a height of 10 yd. Determine the surface area of the pyramid to the nearest square yard.

11. Julie is constructing a tent in the shape of a right square pyramid. She uses 4 poles, each 2.1 m long, for the edges that form the triangular surfaces. The side length of the base of the tent is 1.5 m.

a) Sketch a diagram of the tent.

b) What is the slant height of the tent to the nearest tenth of a metre?

c) What is the lateral surface area of the tent to the nearest square metre?

12. A regular tetrahedron has edge length 10 in.

a) What is the slant height of the tetrahedron to the nearest tenth of an inch?

b) What is the surface area of the tetrahedron to the nearest square inch?

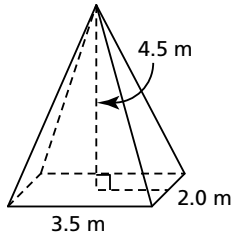
13. An ice-cream cone is to be coated with chocolate on the inside. The cone has an interior diameter of 7.5 cm and an interior height of 10.0 cm. What is the area to be coated? Write the answer to the nearest tenth of a square unit.

14. The Summerhill Pyramid Winery in Kelowna, B.C., has a pyramid that is a replica of the Great Pyramid in Egypt. The Summerhill pyramid has base side length 60 ft. and height 38 ft. The pyramid is to be coated with polished white limestone. What area of limestone is needed to the nearest square foot?

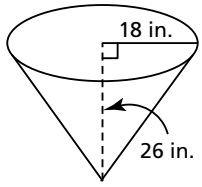
1.5

15. Determine the volume of each object to the nearest cubic unit.

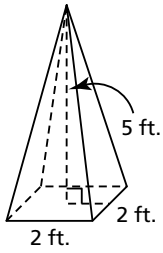
a) right rectangular pyramid



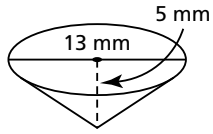
b) right cone



c) right square pyramid



d) right cone



16. To determine the volume of a cone, Owen measured its slant height as 7.3 cm and its base diameter as 9.6 cm. Can Owen determine the volume of the cone with only these measurements? If your answer is yes, show your solution. If your answer is no, explain what Owen needs to do to determine the volume, then calculate the volume.

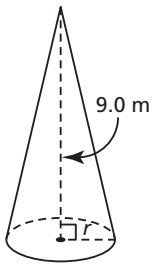
17. Emma used water displacement in a large measuring cylinder to determine that the volume of a right square pyramid was 400 cm^3. Emma measured the side of the base as 10 cm. What was the height of the pyramid?

18. **a)** A solid iron garden ornament has the shape of a right square pyramid. The slant height of the pyramid is 8 in. and the side length of the base is 3 in. Determine the volume of the garden ornament to the nearest cubic inch.

b) Another garden ornament has volume 96 cubic inches. It has the same shape and the same height as the ornament in part a. What is the side length of its base to the nearest inch?

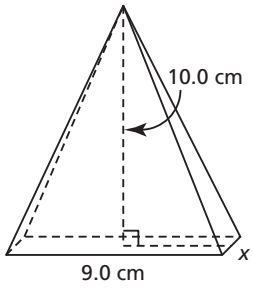
19. For each object, its volume, V, and some dimensions are given. Calculate the dimension indicated by the variable. Write each answer to the nearest tenth of a unit.

a) right cone



$V = 41.6 \text{ m}^3$

b) right rectangular pyramid

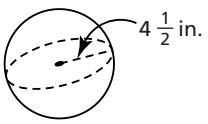


$V = 68.4 \text{ cm}^3$

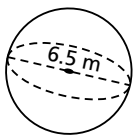
1.6

20. Determine the surface area and volume of each sphere. Write the answers to the nearest whole unit.

a)



b)



21. Sketch a hemisphere with diameter 18 ft.

a) What is the surface area of the hemisphere to the nearest square foot?

b) What is the volume of the hemisphere to the nearest cubic foot?

22. The surface area of a sphere is approximately 66 square inches. What is the diameter of the sphere to the nearest tenth of an inch?

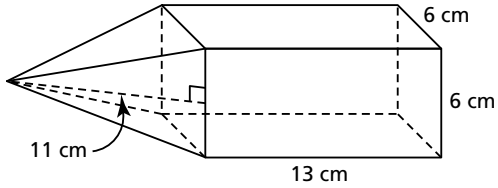
23. A handful of snow is compressed into a spherical snowball. The snowball has circumference 18 cm. What is its volume?

24. A "gazing ball" is a spherical garden ornament with a mirrored surface that reflects its surroundings. The surface area of the ball is approximately 314 square inches. What is its volume to the nearest cubic inch?

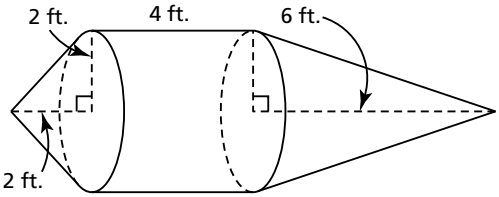
1.7

25. Determine the surface area and volume of each composite object to the nearest whole unit.

a) right square prism and right square pyramid



b) right cylinder and right cones



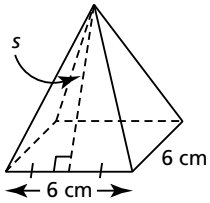
26. A sandcastle comprises a right rectangular prism with base dimensions 75 cm by 50 cm, and height 30 cm. There are 4 congruent cones on the top surface of the prism. Each cone has base diameter 10 cm and slant height 15 cm.

a) Determine the volume of sand required to construct this castle. Write the answer to the nearest cubic centimetre.

b) Determine the surface area of the castle. Write the answer to the nearest square centimetre.

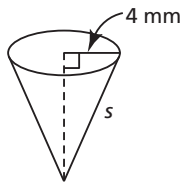
27. For each object, its surface area, SA, and some dimensions are given. Calculate the dimension indicated by the variable. Write each answer to the nearest whole unit.

a) right square pyramid



$SA = 132 \text{ cm}^2$

b) right cone



$SA = 176 \text{ mm}^2$

PRACTICE TEST

For questions 1 and 2, choose the correct answer: A, B, C, or D

1. Which expression represents an approximate measure of 3 cm in inches?

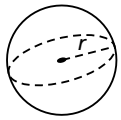
A. $\frac{3}{0.4}$ **B.** $3(0.4)$

C. $\frac{0.4}{3}$ **D.** $\frac{1}{3(0.4)}$

2. Which expression represents the volume of a sphere with radius r?

A. $\frac{4}{3}\pi^3 r$ **B.** $4\pi r^2$

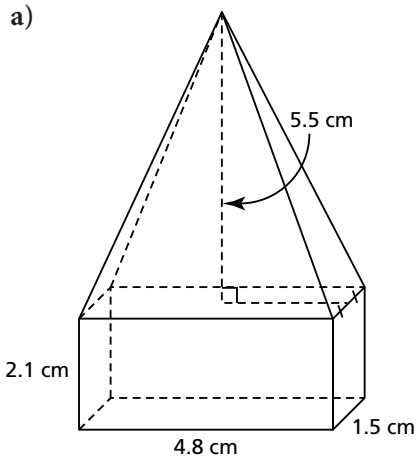
C. $\frac{4}{3}\pi r^3$ **D.** $\frac{4}{3}\pi r^2$



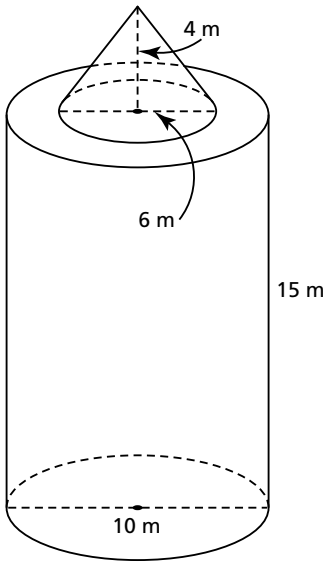
3. A right cylinder and a right cone have the same base radius and the same height. How are their volumes related?

4. Determine the volume and surface area of each composite object. Write your answers to the nearest tenth of a unit.

a)



b)



5. A student measured the height of a right square pyramid as 3 in. and the side length of its base as 2 in.

a) Which measuring instrument do you think the student used? Justify your answer.

b) Explain how the student could use these measurements to determine the volume and surface area of the pyramid.

6. A sphere has a radius of 5.0 cm. What is the radius of a hemisphere that has the same surface area as the sphere? Write the answer to the nearest tenth of a centimetre.