UNIT 4

Linear Relations

How do you think music sales have changed over the past 10 years? 20 years?
In what format do you buy the music you listen to?
In what format did your parents buy the music they listened to as students?
Why might record companies be interested in keeping track of these data?

What You'll Learn

- Use expressions and equations to generalize patterns.
- Verify a pattern by using substitution.
- Graph and analyze linear relations.
- Interpolate and extrapolate to solve problems.

Why It's Important

Patterns and relationships are an important part of math. We can model many real-world situations with a linear relation, and use the relation to make predictions and solve problems. For example, the total cost of a pizza is a fixed cost for a particular size, plus a cost that depends on the number of toppings added.

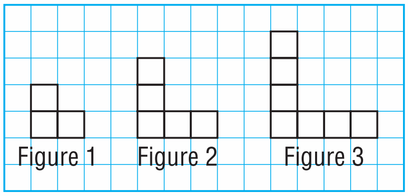
Key Words

- dependent variable
- independent variable
- relation
- linear relation
- interpolation
- extrapolation

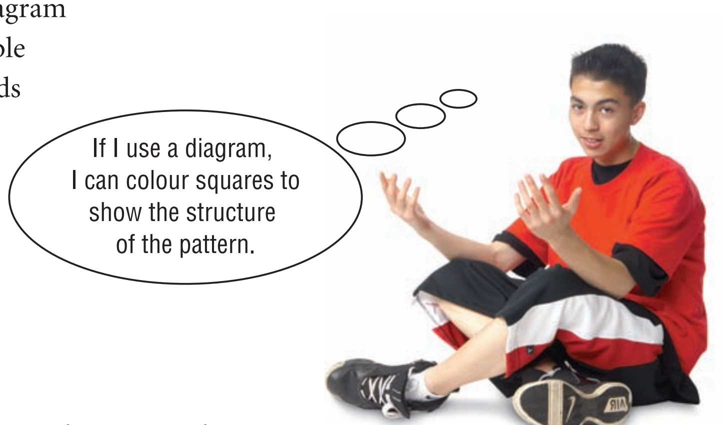
Start Where You Are

How Can I Explain My Thinking?

The pattern of figures below continues. Suppose I have to determine a rule for the number of squares in any figure n in this pattern.



What tools can I use to explain my thinking?
- a diagram
- a table
- words



I might see the pattern this way.

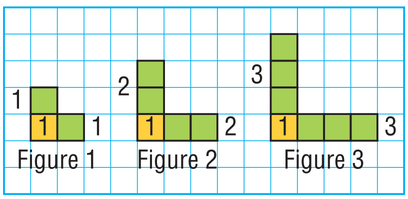
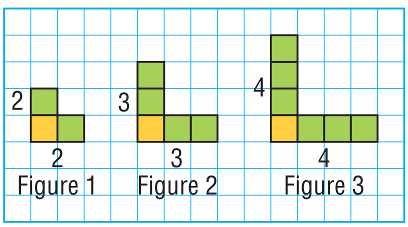


Figure 1: $1 + 1 + 1 = 1 + 2(1)$
Figure 2: $1 + 2 + 2 = 1 + 2(2)$
Figure 3: $1 + 3 + 3 = 1 + 2(3)$
Figure n: $1 + n + n = 1 + 2(n)$

I might also see the pattern this way.



I am counting the yellow square twice, so I will have to subtract 1.

Figure 1: $2 + 2 - 1 = (1 + 1) + (1 + 1) - 1$
Figure 2: $3 + 3 - 1 = (2 + 1) + (2 + 1) - 1$
Figure 3: $4 + 4 - 1 = (3 + 1) + (3 + 1) - 1$
Figure n: $(n + 1) + (n + 1) - 1$

I might also see the pattern a third way.

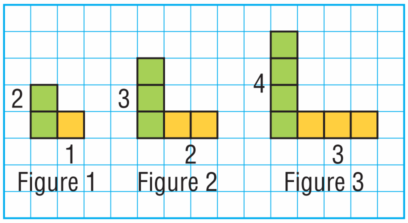


Figure 1: $2 + 1 = (1 + 1) + 1$
Figure 2: $3 + 2 = (2 + 1) + 2$
Figure 3: $4 + 3 = (3 + 1) + 3$
Figure n: $(n + 1) + n$

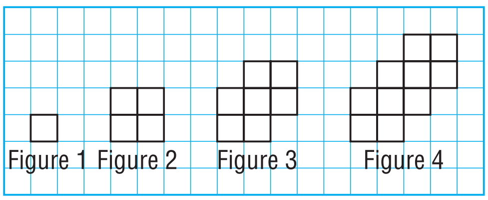
Figure Number	Number of Squares	Pattern
1	3	$2(1) + 1$
2	5	$2(2) + 1$
3	7	$2(3) + 1$
n		$2n + 1$

With each figure, I add 2 extra squares.
The numbers of squares are odd numbers.
Each number is 1 more than a multiple of 2.
So, figure n has $2n + 1$ squares.

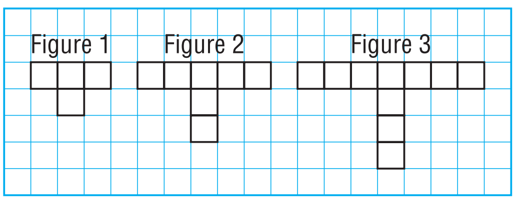
Check

Use the tools *you* find most helpful to determine a rule for the number of squares in figure n of each pattern.

1.



2.



4.1 Writing Equations to Describe Patterns

FOCUS

- Use equations to describe and solve problems involving patterns.

Here is a pattern made from square tiles.

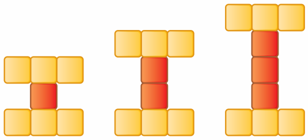


Figure 1 Figure 2 Figure 3

What stays the same in each figure? What changes?
How can we determine the number of square tiles in any figure in the pattern?

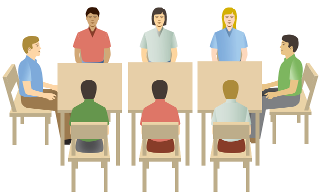
Investigate

2

A banquet hall has small square tables that seat 1 person on each side.
The tables can be pushed together to form longer tables.

1 table

2 tables



3 tables

The pattern continues.

➤ Sketch the next 2 table arrangements in the pattern.
What stays the same in each arrangement? What changes?

➤ What different strategies can you use to determine the number of people at 6 tables? At 10 tables? At 25 tables?

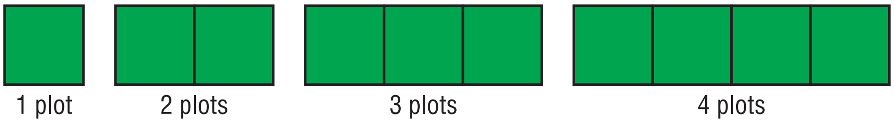
Reflect & Share

Compare your strategies and results with those of your classmates.
If you used different strategies, explain your strategies.
If you did not write an equation, work together to determine an equation that relates the number of people to the number of tables.
Use the equation to determine:

- the number of people at 30 tables
- the number of tables needed to seat 30 people

Connect

A landscape designer uses wooden boards as edging for the plots in a herb garden.



The number of boards, b, is *related* to the number of plots, p.
This relationship can be represented in different ways:

- using pictures
- using a table of values
- using an equation

Here are 2 ways to determine the equation.

➤ Determine a pattern in the number of boards.

	Number of Plots, p	Number of Boards, b	
	1	4	
+1	2	7	+3
+1	3	10	+3
+1	4	13	+3

As the number of plots increases by 1, the number of boards increases by 3. Repeated addition of 3 is the same as multiplication by 3. This suggests that the number of boards may be 3 times the number of plots. So, the equation $b = 3p$ may represent this relationship.

Check whether the equation $b = 3p$ is correct.
When $p = 1$,

$$\begin{aligned} b &= 3(1) \\ &= 3 \end{aligned}$$

This is 1 less than the number 4 in the table.
So, we add 1 to $3p$ to describe the number of boards correctly.

Number of Plots, p	Number of Boards, b
1	$3(1) + 1 = 4$
2	$3(2) + 1 = 7$
3	$3(3) + 1 = 10$
4	$3(4) + 1 = 13$

The terms $3p + 1$ form an *expression* that represents the number of boards for any number of plots p.
An equation is: $b = 3p + 1$

We verify the equation by substituting values of p and b from the table.
For example, check by substituting $p = 4$ and $b = 13$ in $b = 3p + 1$.

Left side: $b = 13$ Right side: $3p + 1 = 3(4) + 1$
$$\begin{aligned} &= 12 + 1 \\ &= 13 \end{aligned}$$

Since the left side equals the right side, the equation is verified.

➤ Determine a pattern in the figures that represent the garden.

Number of Plots, p	Pattern in the Number of Boards	Number of Boards, b
1	1 + 3	1 + 1(3)
2	1 + 3 + 3	1 + 2(3)
3	1 + 3 + 3 + 3	1 + 3(3)
4	1 + 3 + 3 + 3 + 3	1 + 4(3)
:		:
p		1 + p(3)

Each garden needs 1 board for the left border and 3 additional boards for each plot.
That is,
Number of boards = 1 + (Number of plots) × 3
As an equation:
$b = 1 + p(3)$
This can be rewritten as:
$b = 1 + 3p$

Addition is commutative, so $1 + 3p = 3p + 1$.
The equation gives a general pattern rule. We say the equation *generalizes* the pattern. We can use the equation to determine the value of any term.

Example 1 Writing an Equation to Represent a Written Pattern

An airplane is cruising at a height of 10 000 m. It descends to land. This table shows the height of the plane every minute after it began its descent. The height of the plane changes at a constant rate.

Time (t minutes)	Height (h metres)
0	10 000
1	9 700
2	9 400
3	9 100
4	8 800

a) Write an expression for the height in terms of the time since the plane began its descent.
b) Write an equation that relates the height of the plane to the time since it began its descent.
c) What is the height of the plane after 15 min?
d) How long after beginning its descent does the plane land?

A Solution

a) When the time increases by 1 min, the height decreases by 300 m.
Add a third column to the table and write the height in terms of time.

Time (t minutes)	Height (h metres)	Height in Terms of Time
0	10 000	10 000 − 0 = 10 000
1	9 700	10 000 − 300(1) = 9700
2	9 400	10 000 − 300(2) = 9400
3	9 100	10 000 − 300(3) = 9100
4	8 800	10 000 − 300(4) = 8800
:		:
t		10 000 − 300(t)

+1 +1 +1 +1 +1 +1

−300 −300 −300 −300 −300 −300

An expression for the height in terms of time is: $10\ 000 - 300t$

b) For an equation that relates height to time, equate the expression in part a to the height, h.
An equation is: $h = 10\ 000 - 300t$

c) To determine the height of the plane after 15 min, substitute $t = 15$ in the equation:

$$\begin{aligned} h &= 10\ 000 - 300t \\ &= 10\ 000 - 300(15) \\ &= 10\ 000 - 4500 \\ &= 5500 \end{aligned}$$

After 15 min, the plane is at a height of 5500 m.

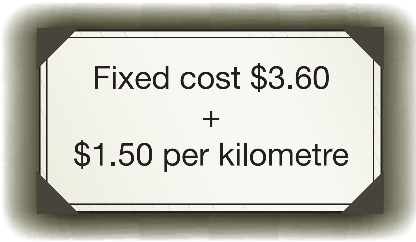
d) When the plane lands, its height is 0.
Substitute $h = 0$ in the equation $h = 10\ 000 - 300t$, then solve for t.

$$\begin{aligned} h &= 10\ 000 - 300t \\ 0 &= 10\ 000 - 300t \\ 300t + 0 &= 10\ 000 - 300t + 300t \\ 300t &= 10\ 000 \\ \frac{300t}{300} &= \frac{10\ 000}{300} \\ t &= 33.\overline{3} \end{aligned}$$

The plane lands about 33 min after beginning its descent.

Example 2 Writing an Equation to Represent an Oral Pattern

I called Kelly's Cabs. The cost of a ride is shown on a poster in the cab.



a) Write an expression for the fare in terms of the fixed cost and the cost per kilometre.
b) Write an equation that relates the fare to the distance travelled.
c) What is the fare for an 11-km ride?

A Solution

a) The fare is $3.60, plus $1.50 per kilometre.
That is, the fare is $3.60 + 1.50 \times$ (distance in kilometres).
Let d represent the distance in kilometres.
So, an expression for the fare is: $3.60 + 1.50 \times d$, or $3.60 + 1.50d$

b) Let F represent the fare in dollars.
Then, an equation that relates F and d is: $F = 3.60 + 1.50d$

c) To determine the cost for an 11-km trip, use the equation: $F = 3.60 + 1.50d$
Substitute: $d = 11$
$F = 3.60 + 1.50(11)$
$ = 3.60 + 16.50$
$ = 20.10$
The fare for an 11-km ride is $20.10.

Discuss the ideas

1. What different ways can you represent a relationship between two quantities?

2. What are the advantages and disadvantages of each way you described in question 1?

3. Suppose you have determined an equation that you think may describe a pattern.
   a) How could you check that your equation is correct?
   b) If you need to adjust the equation, how can you determine what needs to be changed?

Practice

Check

4. In each equation, determine the value of P when $n = 1$.

a) $P = 2n$ **b)** $P = 3n$ **c)** $P = 4n$ **d)** $P = 5n$

5. In each equation, determine the value of A when $n = 2$.

a) $A = 3n + 1$ **b)** $A = 3n + 2$
c) $A = 3n + 3$ **d)** $A = 3n + 4$

6. In a table of values for a pattern, $P = 3$ when $n = 1$; which of the following equations might represent the pattern?

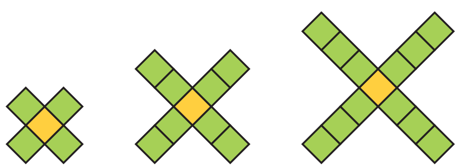
a) $P = 3n$ **b)** $P = n + 3$
c) $P = 2n + 1$ **d)** $P = 3 - n$

7. The pattern in this table continues. Which expression below represents the number of squares in terms of the figure number?

Figure, f	Number of Squares, s
1	6
2	7
3	8
4	9
5	10

a) $5f$ **b)** $2f$ **c)** $f + 5$ **d)** $s + 5$

8. This pattern of squares continues. Which equation below relates the number of squares, n, in a picture to the size number, s?



Size 1 Size 2 Size 3

a) $n = s + 4$ **b)** $n = 4s$
c) $n = 4s + 1$ **d)** $s = 4n$

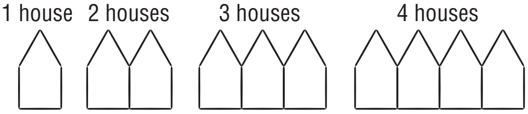
9. The pattern in this table continues. Which equation below relates the number of squares to the figure number?

Figure, f	Number of Squares, s
1	5
2	7
3	9
4	11
5	13

a) $s = 4f + 1$ **b)** $s = 2f + 3$
c) $s = f + 2$ **d)** $f = 2s + 3$

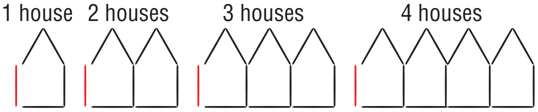
10. Here is a pattern made with toothpicks. The pattern continues.

1 house 2 houses 3 houses 4 houses



Here are the toothpicks rearranged to show what stays the same and what changes in each picture.

1 house 2 houses 3 houses 4 houses



$1 + 1(4)$ $1 + 2(4)$ $1 + 3(4)$ $1 + 4(4)$

a) Explain how the numbers in the expression below each picture describe the arrangement of toothpicks in the picture.

b) Let n represent the number of houses in a picture. Write an expression for the number of toothpicks in n houses.

c) Write an equation that relates the number of toothpicks, t, to n.

d) Verify the equation by showing that it produces the correct number of toothpicks for the first four pictures in the pattern.

Apply

11. The pattern in each table below continues.
For each table:
i) Describe the pattern that relates v to t.
ii) Write an expression for v in terms of t.
iii) Write an equation that relates v to t.
iv) Verify your equation by substituting values from the table.

a)

Term Number, t	Term Value, v
1	11
2	22
3	33
4	44

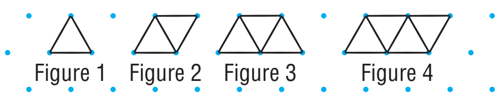
b)

Term Number, t	Term Value, v
1	5
2	8
3	11
4	14

c)

Term Number, t	Term Value, v
1	7
2	6
3	5
4	4

12. Here is a pattern of triangles made with congruent toothpicks.
The pattern continues.



a) Make a table of values for the figure number and the number of toothpicks in a figure. What patterns do you see?
b) Write an expression for the number of toothpicks, t, in figure n.
c) Determine the number of toothpicks in figure 45.
d) Write an equation that relates t to n.
e) Which figure has 17 toothpicks?
How could you check your answer?

13. **Assessment Focus** Hexagonal tables are arranged as shown. The pattern continues. One person sits at each side of a table.

a) Determine the number of people who can be seated at each table arrangement. Record your results in a table.
b) Describe the patterns in the table.
c) What strategies can you use to determine the number of people who could be seated at any table arrangement in the pattern?
d) Write an equation that relates the number of people, p, who can be seated at n tables. How can you check that your equation is correct?
e) How many tables are needed to seat 41 people? How could you check your answer?

Show your work.

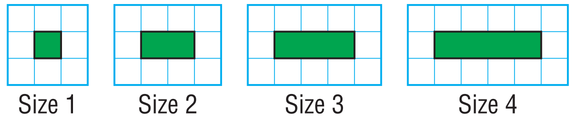
14. The cost to print brochures is the sum of a fixed cost of $250, plus $1.25 per brochure.
a) Write an equation that relates the total cost, C dollars, to the number of brochures, n.
b) What is the cost of printing 2500 brochures?
c) How many brochures can be printed for $625?

Justify your answers.

15. A pizza with tomato sauce and cheese costs \$9.00.
Each additional topping costs \$0.75.
   a) Create a table that shows the costs of a pizza for up to 5 toppings.
   b) Write an equation that relates the cost, C dollars, to the number of toppings, n. Verify your equation by substituting values of n from the table.
   c) Suppose a pizza costs \$15.00. How many toppings were ordered? What strategy did you use? Try a different strategy to check your answer.

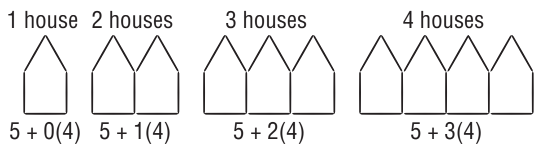
16. Clint has a window cleaning service. He charges a fixed cost of \$12, plus \$1.50 per window.
   a) Write an equation that relates the total cost to the number of windows cleaned. How do you know that your equation is correct?
   b) Clint charged \$28.50 for a job. How many windows did he clean? How do you know?

17. A landscaper uses square patio stones as a border around a rectangular garden.
The number of patio stones needed depends on the size of the garden.
This pattern continues.



The landscaper uses 152 stones. What size of garden does she make?
How can you check your answer?

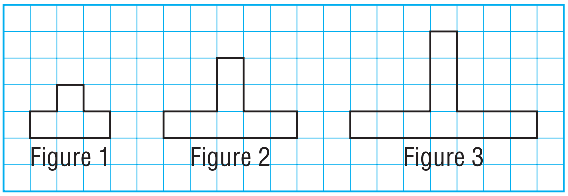
18. Here is another way to rearrange the toothpicks in question 10.



   a) Explain how the expression below each picture describes the number of toothpicks in the picture.
   b) Suppose n represents the number of houses. Write an equation that relates the number of toothpicks, t, to the number of houses, n.
   c) Compare the equation in part b with the equation in question 10c. How can two different equations represent the same pattern? Explain.

Take It Further

19. Here is a pattern of squares. Each square has side length 1 cm. The pattern continues.



a) Make a table that shows each figure number, its perimeter, and its area.

b) Write an equation that can be used to determine the perimeter of any figure in the pattern. Verify the equation. How did you do this?

c) Write an equation that can be used to determine the area of any figure in the pattern. Verify the equation.

d) Determine the perimeter and area of figure 50.

e) Which figure has a perimeter of 100 cm?

f) Which figure has an area of 100 cm^2?

20. The pattern in this table continues.

Term Number, *t*	Term Value, *v*
1	80
2	76
3	72
4	68

a) Write an equation that relates v to t.

b) Verify your equation by substituting values from the table.

21. Marcel has a sheet of paper. He cuts the paper in half to produce two pieces. Marcel places one piece on top of the other. He then cuts these pieces in half. The pattern continues. The table below shows some of Marcel's results.

Number of Cuts	1	2	3	4	5	6	7	8	9	10
Number of Pieces	2	4	8							

a) Copy and complete the table.

b) What patterns do you see in the numbers of pieces?

c) Determine the number of pieces after 15 cuts.

d) Write an equation that relates the number of pieces, P, to the number of cuts, n.

e) How many cuts have to be made to get more than 50 000 pieces? Explain how you found out.

Reflect

Describe some different ways to represent a pattern.
Which way do you prefer when you use a pattern to solve a problem?
Explain your choice.

Tables of Values and Graphing

FOCUS

- Use a spreadsheet to create a table of values and a graph.

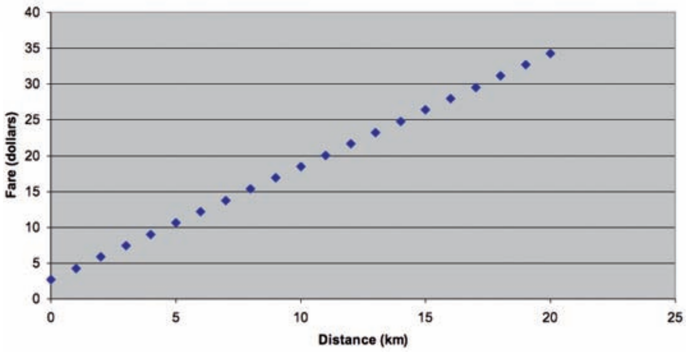
A spreadsheet can be used to create a table of values or to graph a relation.

A taxi company charges a fixed cost of \$2.70, plus \$1.58 per kilometre travelled.

To create a table of values, first write an equation.
Let d represent the distance travelled in kilometres and F represent the fare in dollars.
Then the equation that relates d and F is: $F = 2.70 + 1.58d$

Generate a table of values.
Graph the data. If you need to, use the Help menu to show you how to do this with your software.

	A	B
1	*d*	*F*
2	0	2.7
3	1	4.28
4	2	5.86
5	3	7.44
6	4	9.02
7	5	10.6
8	6	12.18
9	7	13.76
10	8	15.34
11	9	16.92
12	10	18.5
13	11	20.08
14	12	21.66
15	13	23.24
16	14	24.82
17	15	26.4
18	16	27.98
19	17	29.56
20	18	31.14
21	19	32.72
22	20	34.3



Your table and graph should look similar to these.

Check

1. A second taxi company charges a fixed cost of \$4.20, plus \$1.46 per kilometre.

a) Write an equation that relates the fare to the distance travelled.

b) Use a spreadsheet to generate a table of values.

c) Use the spreadsheet to graph the equation.

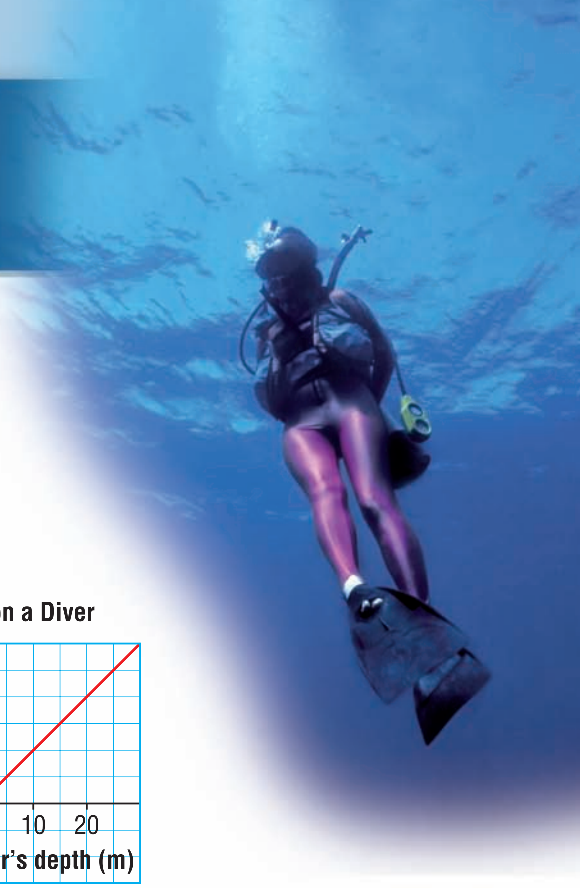
2. In Lesson 4.1, you solved problems involving equations. Choose two questions from *Practice*. For each question:

a) Use a spreadsheet to generate a table of values and solve the problem.

b) Use the spreadsheet to graph the equation.

4.2 Linear Relations

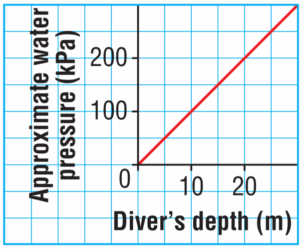
FOCUS
- Analyze the graph of a linear relation.



When a scuba diver goes under water, the weight of the water exerts pressure on the diver.

Diver's Depth (m)	Approximate Water Pressure (kiloPascals)
0	0
5	50
10	100
15	150
20	200

Pressure on a Diver



What patterns do you see in the table and in the graph?
What do these patterns tell you about the relationship between depth and water pressure?

Investigate

2

A local phone company offers a cell phone plan that has a fixed cost per month and a cost related to the number of text messages sent. The fixed cost is $20 and each message sent costs 10¢.

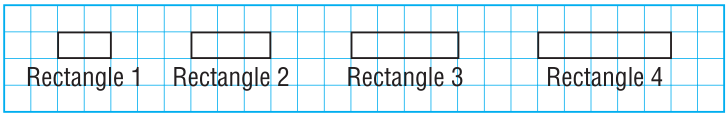
Represent the relation between the total cost and the number of text messages sent, as many different ways as you can.

Reflect & Share

Compare your representations with those of another pair of students. Did you use the same way to represent the pattern? If your patterns are different, explain your pattern to the other students.
If you represented the relation in a different way from your classmates, explain your way to them.

Connect

The first 4 rectangles in a pattern are shown below. The pattern continues.
Each small square has side length 1 cm.



The perimeter of a rectangle is related to the rectangle number.
We can use words, a table, a graph, and an equation to represent this relationship.
Each representation tells us about the relationship between the rectangle number and its perimeter.

In Words

Rectangle 1 has perimeter 6 cm; then, as the rectangle number increases by 1, its perimeter increases by 2 cm.

In a Table

Rectangle Number, n	Perimeter, P (cm)
1	$6 = 2(1) + 4$
2	$8 = 2(2) + 4$
3	$10 = 2(3) + 4$
4	$12 = 2(4) + 4$

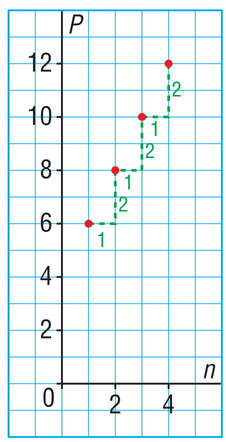
As the rectangle number increases by 1, the perimeter increases by 2 cm.

In a Graph

The graph also shows the pattern. After the first point, each point on the graph is 1 unit right and 2 units up from the preceding point. If we place a transparent ruler on the points, we see that they lie on a straight line.

We do not join the points because the data are discrete.

Graph of P against n



In an Equation

For rectangle n, the perimeter will be $2n + 4$.
The equation is: $P = 2n + 4$
The equation tells us that we can calculate the perimeter of any rectangle in the pattern by multiplying the rectangle number by 2, then adding 4.
The value of the variable P *depends* on the value of the variable n.
We say that P is the **dependent variable** and we plot it on the vertical axis.
The **independent variable** n is plotted on the horizontal axis.
When two variables are related, we have a **relation**.

> **Linear Relation**
> When the graph of the relation is a straight line, we have a **linear relation**.
> In a linear relation, a constant change in one quantity produces a constant change in the related quantity.

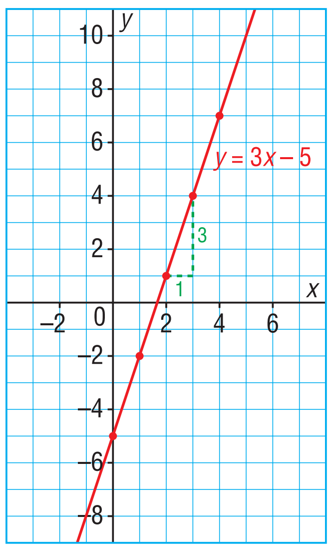
In the relation above, a constant change of 1 in n produced a constant change of 2 cm in P.

Here is the equation of a linear relation: $y = 3x - 5$
x is the independent variable and it is plotted on the horizontal axis.
y is the dependent variable and it is plotted on the vertical axis.
Here are the table and graph that represent this equation.

x	y
0	−5
1	−2
2	1
3	4
4	7

+1 +1 +1 +1

+3 +3 +3 +3



Write the equation on the grid.

When x increases by 1, y increases by 3. This is shown in the table and on the graph.
Since the points lie on a straight line, the equation $y = 3x - 5$ represents a linear relation.
Since we are not told that the data are discrete, we join the points with a line.

Example 1 Graphing a Linear Relation from a Table of Values

The table of values shows the cost of renting DVDs at an online store.

a) Graph the data. Does it make sense to join the points on the graph? Explain.

b) Is the relation linear? Justify your answer.

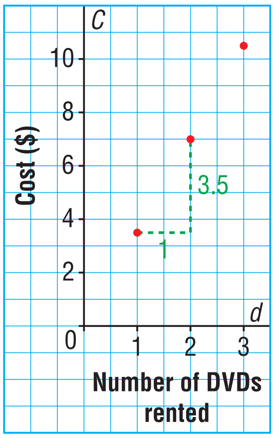
c) Use the table to describe the pattern in the rental costs. How is this pattern shown in the graph?

Number of DVDs Rented, *d*	Cost, *C* ($)
1	3.50
2	7.00
3	10.50
4	14.00
5	17.50

A Solution

a) Plot the points on a grid.

Cost to Rent DVDs



Since the cost depends on the number of DVDs rented, plot *d* horizontally and *C* vertically.

The number of DVDs rented is a whole number. We cannot rent 1.5 DVDs or any other fractional number of DVDs. So, it does not make sense to join the points.

b) The points on the graph lie on a straight line, so the relation is linear.

c) As the number of DVDs rented increases by 1, the rental cost increases by \$3.50. Each point on the graph is 1 unit right and 3.5 units up from the previous point. The pattern of increases in the table produces a graph that is a straight line.

Example 2 Graphing a Linear Relation from an Equation

A relation has the equation: $y = 6 - 3x$

a) Create a table of values for the relation for values of x from -3 to 3.

b) Graph the relation. Does it make sense to join the points on the graph? Explain.

c) What patterns are in the graph? How do these patterns relate to the table of values?

d) Is the relation linear? Justify your answer.

Solutions

Method 1

a), b) To create a table of values, substitute the given values of x in the equation:

$y = 6 - 3x$

Substitute: $x = -3$
$y = 6 - 3(-3)$
$= 6 + 9$
$= 15$

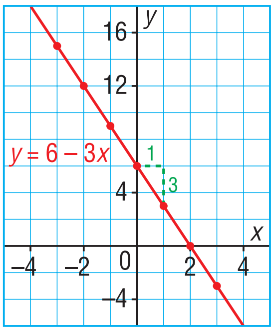
Substitute: $x = -2$
$y = 6 - 3(-2)$
$= 6 + 6$
$= 12$

Substitute: $x = -1$
$y = 6 - 3(-1)$
$= 6 + 3$
$= 9$

Substitute: $x = 0$
$y = 6 - 3(0)$
$= 6 - 0$
$= 6$

Use mental math to repeat the above process for $x = 1$, $x = 2$, and $x = 3$.
Write the values of x and y in a table.

x	y
−3	15
−2	12
−1	9
0	6
1	3
2	0
3	−3



Since the data are not discrete, join the points to form a line.

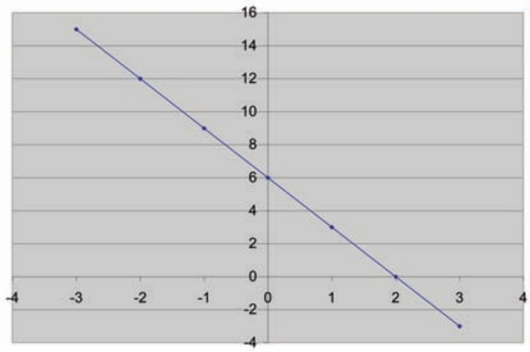
Method 2

Use a spreadsheet.

a) Input the equation and make a table.

	A	B
1	x	y
2	-3	15
3	-2	12
4	-1	9
5	0	6
6	1	3
7	2	0
8	3	-3

b) Highlight the table.
Graph the data.



c) On the graph, to get from one point to the next, move 1 unit right and 3 units down.
In the table, when x increases by 1, y decreases by 3.

d) The relation is linear because its graph is a straight line.

Example 3 Solving Problems Using a Linear Relation

The student council is planning to hold a dance. The profit in dollars is 4 times the number of students who attend, minus \$200 for the cost of the music.

a) Write an equation that relates the profit to the number of students who attend.

b) Create a table of values for this relation.

c) Graph the data in the table. Does it make sense to join the points? Explain.

d) How many students have to attend to make a profit?

A Solution

a) Profit in dollars = 4 × number of students who attend − 200
Choose variables to represent the numbers that change.
Let n represent the number of students who attend.
Let P represent the profit in dollars.
An equation is: $P = 4n - 200$

b) Choose 3 values for n, then calculate the corresponding values of P.
Use the equation: $P = 4n - 200$

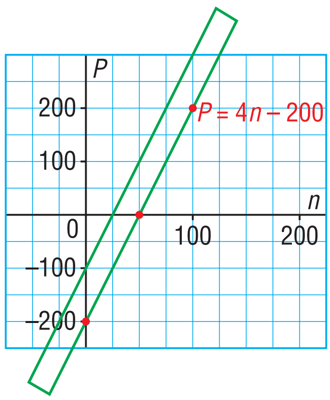
Substitute: $n = 0$
$P = 4(0) - 200$
$= 0 - 200$
$= -200$

Substitute: $n = 50$
$P = 4(50) - 200$
$= 200 - 200$
$= 0$

Substitute: $n = 100$
$P = 4(100) - 200$
$= 400 - 200$
$= 200$

n	P
0	−200
50	0
100	200

c) Plot the points on a grid.



A straightedge verifies that the points lie on a straight line.
Some values between the plotted points are permitted, but not others.
For example, there could be 82 students attending the dance, but not 82.5.
So, the points are not joined.

d) When P is negative, a loss is made.
When $P = 0$, $n = 50$, and the profit is 0.
When $P > 0$, $n > 50$, and there is a profit.
So, 51 or more students have to attend before a profit can be made.

Discuss the ideas

1. a) How do you know whether a graph represents a linear relation?
   b) How do you know whether a table of values represents a linear relation?

2. a) How many points do you need to graph a line?
   b) Why do we often use 3 points? Should we use more points? Explain.

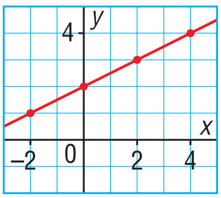
3. How do you know when to connect the points on a graph?

Practice

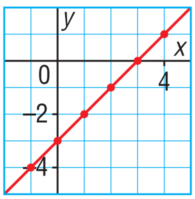
Check

4. Which graphs represent a linear relation? How do you know?

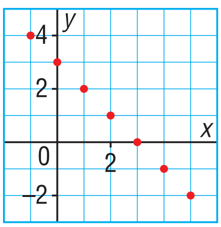
a)



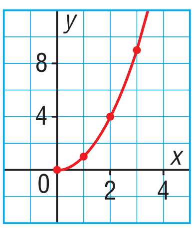
b)



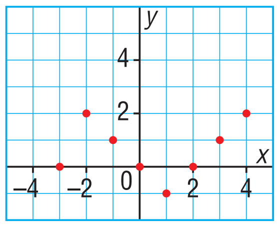
c)



d)



e)



Apply

5. For each table of values below:
   i) Does it represent a linear relation?
   ii) If the relation is linear, describe it.
   iii) If the relation is not linear, explain how you know.

a)

x	y
1	4
2	13
3	22
4	31
5	40

b)

x	y
9	8
8	11
7	14
6	17
5	20

c)

x	y
0	0
1	2
2	6
3	12
4	20

d)

x	y
1	3
4	5
7	7
10	9
13	11

6. Graph the linear relations you identified in question 5. How does each graph verify your answers to question 5?

7. Copy and complete each table of values.

a) $y = 2x$

x	y
1	
2	
3	
4	

b) $y = x + 2$

x	y
1	
2	
3	
4	

c) $y = -2x$

x	y
2	
4	
6	
8	

d) $y = x - 2$

x	y
4	
5	
6	
7	

8. Here is a partially completed table of values for a linear relation.

x	2	3	4	5	6	7	8
y				15	18		

a) Determine the missing values of y. Explain how you found these values.

b) Describe the patterns in the table.

c) Write an equation that represents the linear relation. How do you know that your equation is correct?

d) Graph the data. How are the patterns you described in part b shown in the graph?

e) Suppose you want to determine the value of y when $x = -1$. How could you use the table and equation to do this? What is the value of y when $x = -1$?

9. Each table of values represents a linear relation. Copy and complete each table. Explain your reasoning.

a)

x	y
2	11
3	14
4	
5	
6	

b)

x	y
1	
3	8
5	9
7	
9	

c)

x	y
−4	
−2	7
0	3
2	
4	

d)

x	y
4	
6	−7
8	−4
10	
12	

10. Create a table of values for each linear relation, then graph the relation. Use values of x from -2 to 2.

a) $y = 3x$

b) $y = x + 3$

c) $y = x - 3$

d) $y = 5 - x$

e) $y = 1 - 4x$

f) $y = -2x - 3$

11. Jin is cycling at an average speed of 4 m/s. He travels a distance, d metres, in t seconds.

a) Write an equation that relates d and t.

b) Create a table of values for this relation.

c) Graph the data. Should you join the points? Explain your reasoning.

d) Is the relation between distance and time linear?

i) How do you know from the table of values?

ii) How you know from the graph?

e) How far does Jin travel in 3.5 h?

f) What time does it take Jin to travel 17 km?

12. In 2008, the Goods and Services Tax (GST) was 5%. To determine the tax, T dollars, charged on a given purchase price, p dollars, multiply the purchase price by 0.05.

a) Write an equation that relates T to p.

b) Copy and complete this table of values.

p	0	10	20	30	40
T					

c) What patterns do you see in the table?

d) Graph the data.
Which variable will you plot on the horizontal axis? Explain your reasoning.

e) Should you connect the points on the graph? Explain.

f) How are the patterns in the table shown in the graph?

13. An amusement park charges an admission fee of \$10, plus \$2 per ride.

a) Choose variables to represent the total cost in dollars and the number of rides that are taken. Write an equation that relates the total cost to the number of rides.

b) Graph the equation.

c) What is the total cost for 7 rides?

d) How many rides can be taken for a total cost of \$38?

14. Assessment Focus Danica is having a party. She estimates that she will need 3 pieces of pizza for each guest invited, and 6 extra pieces in case someone shows up unexpectedly.

a) Explain why this situation can be represented by the equation $P = 3n + 6$. What do P and n represent in the equation?

b) Make a table of values for the relation.

c) Graph the data. Will you join the points on the graph? Explain.

d) Is the relation linear?

i) How do you know from the table of values?

ii) How do you know from the graph?

e) If the relation is linear, explain what this means in the context of this situation.

15. A small plane is at a height of 1800 m when it starts descending to land.
The plane's height changes at an average rate of 150 m per minute.

a) Choose variables to represent the height in metres and the time in minutes since the plane began its descent. Write an equation that relates the height to the time.

b) Graph the equation.

c) What is the height of the plane 6 min after it began its descent?

d) When is the plane 100 m above the ground?

16. Jada rollerblades from Regina to Saskatoon to raise funds for cancer research. The trip is 250 km. Jada estimates that she can rollerblade at an average speed of 8 km/h.

a) Choose variables to represent the time Jada has travelled in hours and the distance in kilometres that she has yet to travel. Write an equation that relates the distance to the time.

b) Graph the equation.

c) How far has Jada still to travel after 12 h?

d) How many hours will it take Jada to complete the trip?

17. Describe a situation that could be represented by each equation.

a) $M = 2n + 5$ **b)** $E = 3.50n$

c) $C = 12 + 5d$ **d)** $H = 100 - 5n$

Take It Further

18. This table of values represents a linear relation. Copy and complete the table. Explain your reasoning.

x	−3	−1	2	5	9	14	20
y	29		23				

Reflect

What does it mean when we say that the relation between two quantities is linear? What patterns are there in the table of values and in the graph of a linear relation? Include examples in your explanation.

Math Link

Science

When an object falls to the ground, it accelerates due to the force of gravity. The relation between the speed of the object and the time it falls is linear.

4.3 Another Form of the Equation for a Linear Relation

FOCUS

- Recognize the equations of horizontal, vertical, and oblique lines, and graph them.

Investigate

2

Suppose you have a piece of ribbon 20 cm long.

➤ How many different ways could you cut it into two pieces?
What are the possible lengths of the two pieces?

➤ How are the lengths of the two pieces related?
Show this relation:
- in words
- in a table
- in a graph
- as an equation

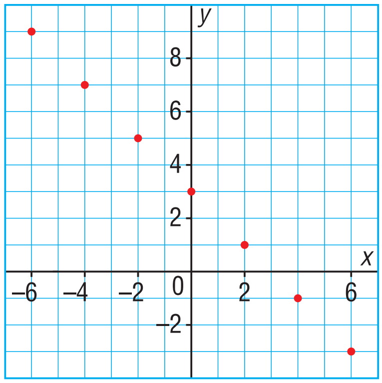
Reflect & Share

Share your different forms of the relation with another pair of students.
If any forms are different, is one of the forms incorrect?
How could you find out?
If you wrote your equation the same way, try to think of a different way to write it.

Connect

Two integers have a sum of 3.
Let x and y represent the two integers.
Here is a table of values and a graph to represent the relation.

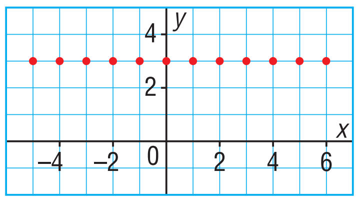
First Integer, x	Second Integer, y
−6	9
−4	7
−2	5
0	3
2	1
4	−1
6	−3



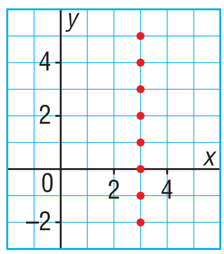
The points lie on a straight line, so the relation is linear.
We can write this linear relation as:
First integer + second integer = 3
Then, the linear relation is: $x + y = 3$
This equation has both variables on the left side of the equation.
It illustrates another way to write the equation of a linear relation.

Suppose one variable does not appear in the equation.

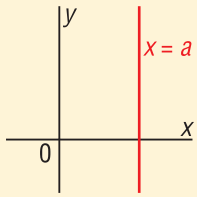
- Suppose x does not appear in $x + y = 3$.
  Then we have the equation $y = 3$.
  To graph this equation on a grid,
  plot points that have a y-coordinate of 3.
  All the points lie on a horizontal line
  that is 3 units above the x-axis.



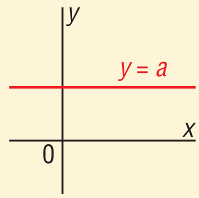
- Suppose y does not appear in $x + y = 3$.
  Then we have the equation $x = 3$.
  To graph this equation on a grid,
  plot points that have an x-coordinate of 3.
  All the points lie on a vertical line that is
  3 units to the right of the y-axis.



The graph of the equation $x = a$, where a is a constant, is a vertical line. Every point on the graph has an x-coordinate of a.



The graph of the equation $y = a$, where a is a constant, is a horizontal line. Every point on the graph has a y-coordinate of a.



Example 1 Graphing and Describing Horizontal and Vertical Lines

For each equation below:

i) Graph the equation.

ii) Describe the graph.

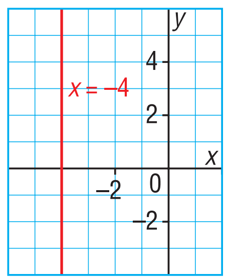
a) $x = -4$ **b)** $y + 2 = 0$ **c)** $2x = 5$

A Solution

a) $x = -4$

i) The x-coordinate of every point on this line is -4.

ii) The graph is a vertical line that intersects the x-axis at -4.



b)

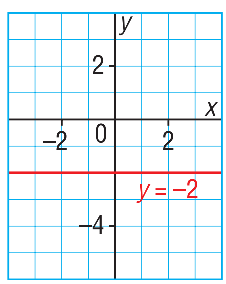
$$y + 2 = 0 \qquad \text{Solve for } y.$$

$$y + 2 - 2 = 0 - 2 \qquad \text{Subtract 2 from each side.}$$

$$y = -2$$

i) The y-coordinate of every point on this line is -2.

ii) The graph is a horizontal line that intersects the y-axis at -2.



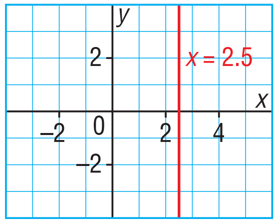
c) $2x = 5$ Solve for x.

$\frac{2x}{2} = \frac{5}{2}$ Divide both sides by 2.

$x = 2.5$

i) The x-coordinate of every point on this line is 2.5.

ii) The graph is a vertical line that intersects the x-axis at 2.5.



Example 2 Graphing an Equation in the Form $ax + by = c$

For the equation $3x - 2y = 6$:

a) Make a table of values for $x = -4$, 0, and 4.

b) Graph the equation.

A Solution

a) $3x - 2y = 6$

Substitute each value of x, then solve for y.

Substitute: $x = -4$

$$\begin{aligned} 3(-4) - 2y &= 6 \\ -12 - 2y &= 6 \\ -2y &= 6 + 12 \\ -2y &= 18 \\ y &= -9 \end{aligned}$$

Substitute: $x = 0$

$$\begin{aligned} 3(0) - 2y &= 6 \\ 0 - 2y &= 6 \\ -2y &= 6 \\ y &= -3 \end{aligned}$$

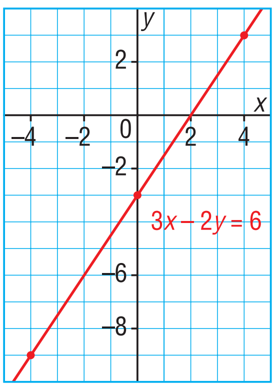
Substitute: $x = 4$

$$\begin{aligned} 3(4) - 2y &= 6 \\ 12 - 2y &= 6 \\ -2y &= 6 - 12 \\ -2y &= -6 \\ y &= 3 \end{aligned}$$

x	y
−4	−9
0	−3
4	3

b) Plot the points on a grid.

Join the points.



Discuss the ideas

1. The graph of an equation such as $3x - 2y = 6$ is a slanted or an *oblique* line. How are the equations for oblique lines different from the equations for horizontal and vertical lines?

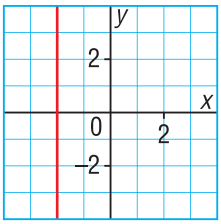
2. Students often mistakenly think that $x = 3$ is a horizontal line instead of a vertical line.
Why might they make this mistake? How might the students reason to avoid making this mistake?

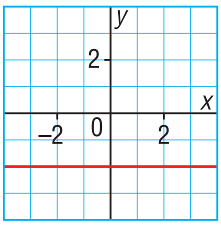
3. How do you recognize the equation of:
a) a vertical line? **b)** a horizontal line?

Practice

Check

4. Which equation describes each graph?
i) $x = -2$ **ii)** $x = 2$
iii) $y = -2$ **iv)** $y = 2$

a)


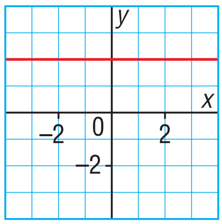
b)


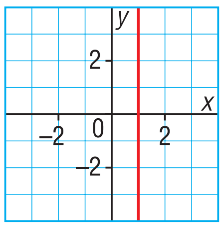
5. Does each equation describe a vertical line, a horizontal line, or an oblique line?
Describe each horizontal and vertical line.
a) $y = 7$ **b)** $x - y = 3$
c) $x = -5$ **d)** $x + 9 = 0$
e) $2y = 5$ **f)** $y = 6 - 2x$

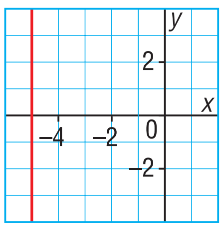
Apply

6. Describe the graph of each line. Graph each line to check your description.
a) $y = 5$ **b)** $x = -1$
c) $x = -5$ **d)** $y = 7$

7. Write an equation to describe each line.

a)


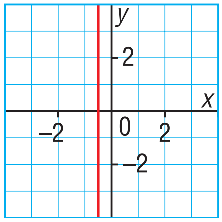
b)


c)


8. Which equation best describes the graph below? Explain your choice.

a) $x - 2 = 0$ b) $2x + 1 = 0$

c) $2y - 1 = 0$ d) $2x - 1 = 0$



9. The sum of two numbers is 15. Let p and q represent the two numbers.

a) Complete a table for 6 different values of p.

b) Graph the data. Should you join the points? Explain.

c) Write an equation that relates p and q.

10. a) For each equation below:

- Make a table of values for $x = -2, 0$, and 2.
- Graph the equation.

i) $x + y = 6$ ii) $x - y = 6$

iii) $x + y = -6$ iv) $x - y = -6$

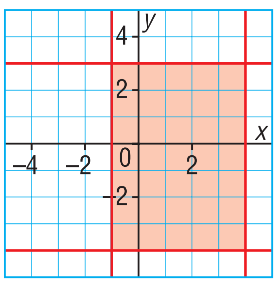
b) How are the graphs in part a alike? How are they different?

11. Graph each line. Explain your work.

a) $y + 3 = -2$ b) $2x = 7$

c) $3x + 1 = -5$ d) $2y - 2 = 10$

12. Write the equations of the lines that intersect to form the shaded rectangle.



13. **Assessment Focus**

a) Graph the following lines on the same grid. What shape do they form?

i) $x = -3$ ii) $y = 2$

iii) $x - 1 = 0$ iv) $y + 2 = 0$

b) Construct a congruent shape on the grid with one of its vertices at the origin.

c) Write the equations of the lines that form the shape you drew.

d) Is there more than one shape you can draw in part b? If your answer is yes, draw any more possible shapes.

Show your work.

14. The distance between Edmonton and Calgary is about 300 km. Kate leaves Calgary to drive to Edmonton. Let t kilometres represent the distance Kate has travelled. Let e kilometres represent the distance she has yet to travel to Edmonton.

a) Copy and complete this table for 6 different values of t.

Distance Travelled, t (km)	Distance to Edmonton, e (km)
0	300

b) What is the greatest value of t that could be in the table? Explain.

c) Graph the data. Should you join the points? Explain.

d) Write an equation that relates t and e.

15. For each equation below:

- Make a table for the given values of x.
- Graph the equation.

a) $2x + y = 6$; for $x = -3, 0, 3$

b) $3x - y = 2$; for $x = -2, 0, 2$

c) $x + 2y = -6$; for $x = -4, 0, 4$

d) $3x - 2y = -6$; for $x = -2, 0, 2$

16. a) On a grid, draw horizontal and vertical lines to construct a shape that satisfies the following conditions:

- The shape is a square with area 9 square units.
- One vertex is at the origin.

b) Write the equations of the lines that form the square.

c) Is it possible to draw another square that satisfies the conditions in part a? If your answer is yes, draw this square and write the equations of the lines that form it.

17. The difference of two numbers is 6. Let a represent the greater number and b the lesser number.

a) Complete a table for 6 different values of a.

b) Graph the data. Should you join the points? Explain.

c) Write an equation that relates b and a.

18. a) Graph these equations on the same grid:

$x = 2 \qquad y = 1 \qquad x + y = 8$

b) What shape is formed by the lines in part a? How do you know?

Take It Further

19. The sum of two rational numbers is $2\frac{1}{2}$.

a) Choose two variables to represent these rational numbers. Make a table to show 5 possible pairs of numbers that satisfy this relation.

b) Graph the data. Describe your graph.

c) Write an equation for the relation.

20. The difference of two rational numbers is -7.5.

a) Choose two variables to represent these rational numbers. Make a table to show 5 possible pairs of numbers that satisfy this relation.

b) Graph the data. Describe your graph.

c) Write an equation for the relation.

21. For each equation below:

- Make a table for 3 values of x.
- Graph the equation.

a) $\frac{1}{2}x + y = 4$

b) $\frac{1}{3}x - y = 2$

c) $\frac{1}{2}x + \frac{1}{3}y = 6$

d) $\frac{1}{3}x - \frac{1}{2}y = -1$

e) $\frac{1}{3}x + \frac{1}{2}y = -3$

f) $\frac{1}{4}x - \frac{1}{2}y = 1$

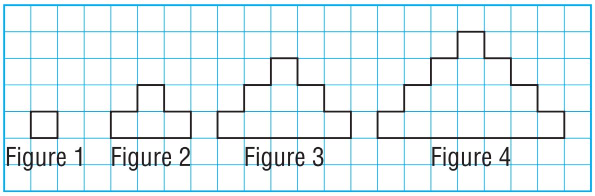
Reflect

How are the equations of horizontal and vertical lines alike?
How are they different?
How can you recognize the equation of each line?

Mid-Unit Review

4.1

1. This pattern of squares continues.



a) Make a table that shows the figure number, n, and the perimeter of a figure, P. What patterns do you see?

b) Write an expression for the perimeter of figure n.

c) What is the perimeter of figure 40?

d) Write an equation that relates P to n.

e) Which figure has a perimeter of 136 units? How do you know?

2. A phone company charges a fixed cost of \$10 per month, plus \$0.25 per minute for long distance calling.

a) Write an equation that relates the monthly cost, C dollars, to t, the time in minutes.

b) In one month, the time for the long distance calls was 55 minutes. What was the monthly cost?

c) For one month, the cost was \$22.50. How many minutes of long distance calls were made?

4.2

3. Create a table of values for each linear relation, then graph the relation. Use values of x from -3 to 3.

a) $y = -3x$ b) $y = 2x$

c) $y = 2 - 4x$ d) $y = -2x + 4$

e) $y = -3 + x$ f) $y = -x + 3$

4. Alicia buys a \$300-jacket on lay away. She made a down payment of \$30 and is paying \$15 per week. The total paid, P dollars, after n weeks can be represented by the equation $P = 15n + 30$.

a) Create a table of values to show the total paid in each of the first 5 weeks.

b) Graph the data. Should you join the points on the graph? Explain.

c) How do the patterns in the graph relate to the patterns in the table?

5. Each table of values represents a linear relation. Copy and complete each table. Explain your reasoning.

a)

x	y
1	10
2	14
3	
4	
5	

b)

x	y
1	
3	-10
5	-14
7	
9	

c)

x	y
-2	
-1	
0	-3
1	3
2	

d)

x	y
2	
4	-2
6	-5
8	
10	

4.3

6. a) Graph each equation.

i) $y = 1$ ii) $x = -4$

iii) $x + y = 8$ iv) $2x - y = 12$

b) For which equations in part a did you *not* need to make a table of values? Explain why.

7. The difference of two numbers is 1. Let g represent the greater number and n the lesser number.

a) Complete a table for 4 different values of n.

b) Graph the data. Should you join the points? Explain.

c) Write an equation that relates n and g.

GAME

What's My Point?

You will need

- grid paper
- a ruler
- a pencil

Number of Players

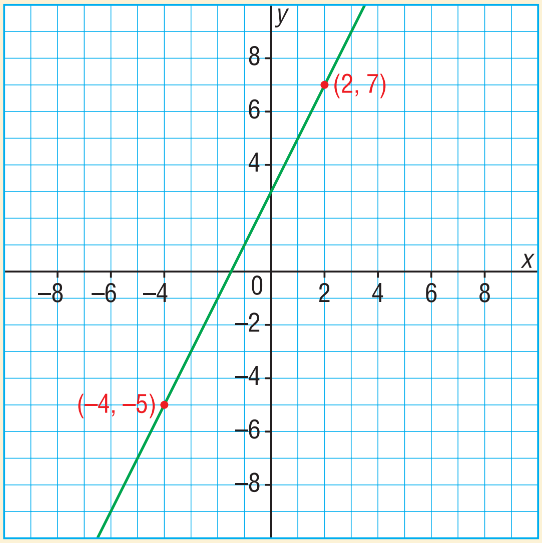
- 2

Goal of the Game

- To have the lesser score at the end of 3 rounds

How to Play

Write the equations of three different linear relations.
Graph each equation on a grid like this.
Plot all points that have integer coordinates.



1. Player A chooses two points on one line.
   She keeps these points secret.
   Player A tells Player B the equation of the line.
   Player B tells the coordinates of a point on the line.
   Player A says whether Player B's point is:
   - one of the chosen points
   - on the line and above the chosen points
   - on the line and below the chosen points, or
   - between the chosen points

   Player B continues to name points on the line until he names both chosen points.
   Each guess counts as 1 point.
2. Player A and B switch roles, with Player A guessing the points selected by Player B.
3. Play continues until all three graphs have been used.
   The player with fewer points wins.
4. Suppose your opponent gave you the equation $y = 5x - 6$.
   Which two points might you guess? Explain.
5. Create a graph that might make it difficult for your opponent to guess your two points.
   Explain why it would be difficult.

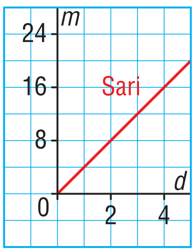
4.4 Matching Equations and Graphs

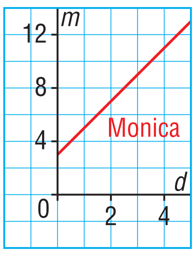
FOCUS

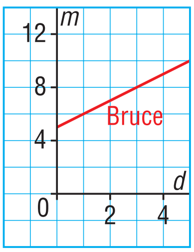
- Match equations and graphs of linear relations.

Investigate

Bruce, Monica, and Sari participate in a 5-km walk for charity.
Each student has a different plan to raise money from her or his sponsors.
These graphs show how the amount of money a sponsor owes is related to the distance walked.







➤ Match each graph with its equation: $m = 2d + 3$ $\quad m = 4d \quad$ $m = d + 5$
Explain your strategy.

➤ Describe each person's sponsorship plan.

Reflect & Share

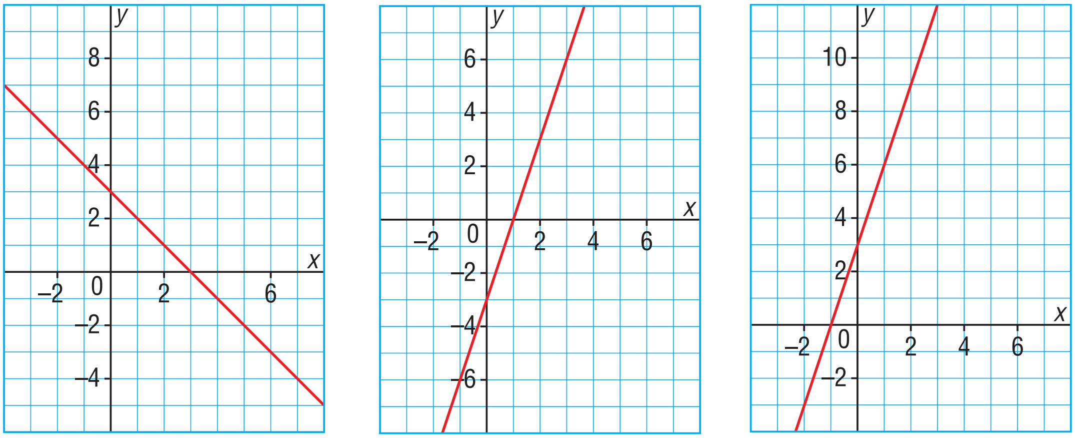
Compare your strategies and descriptions with those of another pair of students.
Did you use the same strategies to match each graph and its equation?
If not, explain your strategies to the other students.

Connect

The 3 graphs below have these equations, but the graphs are not in order:

$y = 3x + 3 \qquad x + y = 3 \qquad y = 3x - 3$

Graph A **Graph B** **Graph C**



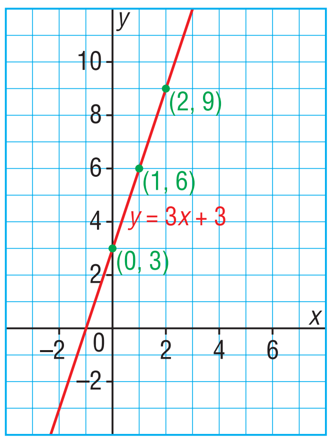
To match each equation with its graph,
use the equation to determine the coordinates of 3 points.
Then find which graph passes through those 3 points.

➤ For $y = 3x + 3$

Substitute: $x = 0$
$y = 3(0) + 3$
$y = 3$
One point is: (0, 3)

Substitute: $x = 1$
$y = 3(1) + 3$
$y = 6$
One point is: (1, 6)

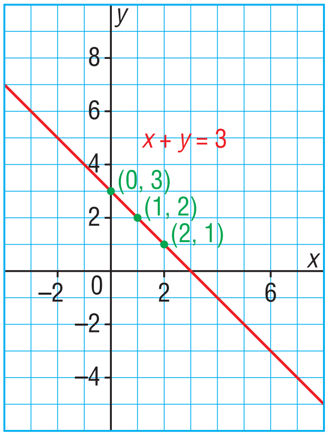
Substitute: $x = 2$
$y = 3(2) + 3$
$y = 9$
One point is: (2, 9)



The graph that passes through these 3 points is Graph C.

➤ For $x + y = 3$

Substitute: $x = 0$
$0 + y = 3$
$y = 3$
One point is: (0, 3)



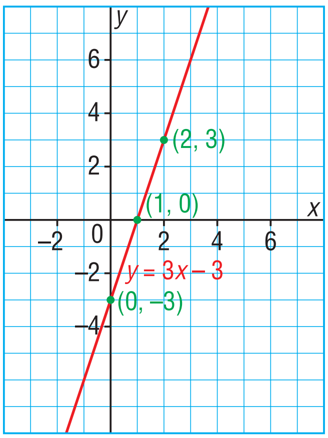
Substitute: $x = 1$
$1 + y = 3$
$y = 2$
One point is: (1, 2)

Substitute: $x = 2$
$2 + y = 3$
$y = 1$
One point is: (2, 1)

The graph that passes through these 3 points is Graph A.

So, the equation $y = 3x - 3$ must match Graph B. Substitute to check.

Substitute: $x = 0$
$y = 3(0) - 3$
$y = -3$
One point is: $(0, -3)$



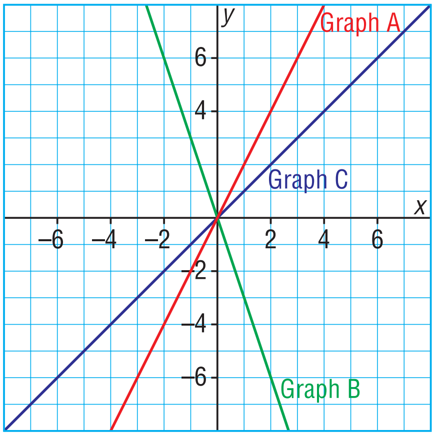
Substitute: $x = 1$
$y = 3(1) - 3$
$y = 0$
One point is: (1, 0)

Substitute: $x = 2$
$y = 3(2) - 3$
$y = 3$
One point is: (2, 3)

The graph that passes through these 3 points is Graph B.

Example 1 Matching Equations with Graphs that Pass through the Origin

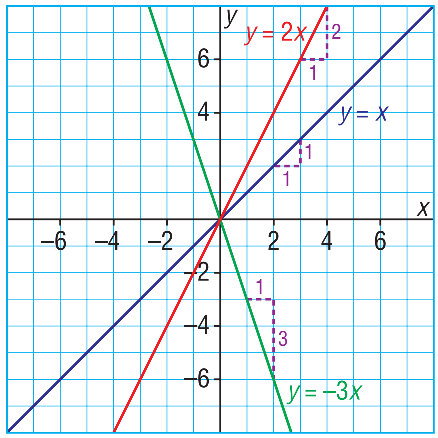
Match each graph on the grid with its equation below.



$y = x$
$y = 2x$
$y = -3x$

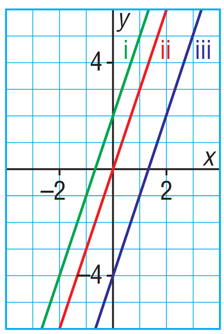
A Solution

Rewrite $y = x$ as $y = 1x$. The coefficient of x represents the pattern of the points on the graph.
In the equation $y = 1x$, the 1 indicates that when x increases by 1 unit, y also increases 1 unit.
This matches Graph C.
In the equation $y = 2x$, the 2 indicates that when x increases by 1 unit, y increases by 2 units.
This matches Graph A.
In the equation $y = -3x$, the -3 tells us that when x increases by 1 unit, y decreases by 3 units.
This matches Graph B.



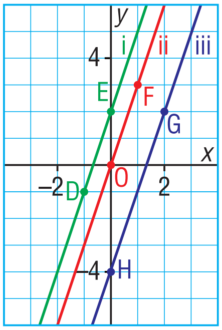
Example 2 Identifying a Graph Given Its Equation

Which graph on this grid has the equation $y = 3x - 4$?
Justify the answer.



A Solution

Pick 2 points on each graph and check to see if their coordinates satisfy the equation.



Two points on Graph i have coordinates
D(−1, −1) and E(0, 2).
Substitute $x = -1$ and $y = -1$ in $y = 3x - 4$.

Left side: $y = -1$ Right side: $3x - 4 = 3(-1) - 4$
$= -7$

The left side does not equal the right side.
So, these coordinates do not satisfy the equation and Graph i does not have equation $y = 3x - 4$.

Two points on Graph ii have coordinates O(0, 0) and F(1, 3).
Substitute $x = 0$ and $y = 0$ in $y = 3x - 4$.

Left side: $y = 0$ Right side: $3x - 4 = 3(0) - 4$
$= -4$

The left side does not equal the right side.
So, these coordinates do not satisfy the equation and Graph ii does not have equation $y = 3x - 4$.

Two points on Graph iii have coordinates G(2, 2) and H(0, −4).
Substitute $x = 2$ and $y = 2$ in $y = 3x - 4$.

Left side: $y = 2$ Right side: $3x - 4 = 3(2) - 4$
$= 2$

The left side does equal the right side, so the coordinates of G satisfy the equation.
Substitute $x = 0$ and $y = -4$ in $y = 3x - 4$.

Left side: $y = -4$ Right side: $3x - 4 = 3(0) - 4$
$= -4$

The left side does equal the right side, so the coordinates of H satisfy the equation.
Since both pairs of coordinates satisfy the equation, Graph iii has equation $y = 3x - 4$.

Discuss the ideas

1. When we match an equation to a graph by determining coordinates of points on the graph, why is it helpful to check 3 points, even though 2 points are enough to identify a line?

2. When we choose points on a graph to substitute their coordinates in an equation, what is an advantage of choosing the points where the graph intersects the axes?

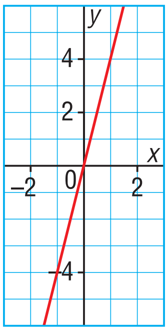
Practice

Check

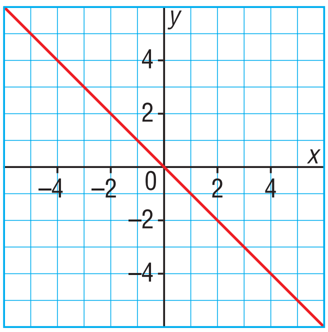
3. Match each equation with a graph below.

a) $y = 2x$ **b)** $y = 4x$ **c)** $y = -x$

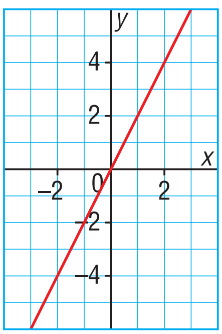
i)



ii)



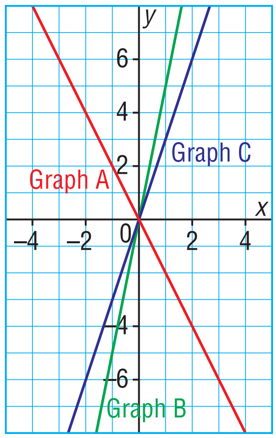
iii)



Apply

4. Match each equation with a graph on the grid below.

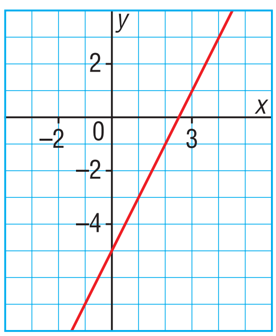
a) $y = 3x$ **b)** $y = 5x$ **c)** $y = -2x$



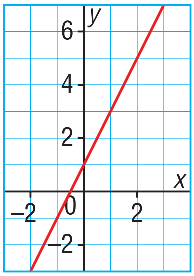
5. Match each equation with a graph below. Which strategy did you use?

a) $y = 2x + 1$ **b)** $y = 2x + 3$ **c)** $y = 2x - 5$

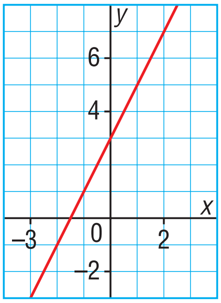
i)



ii)



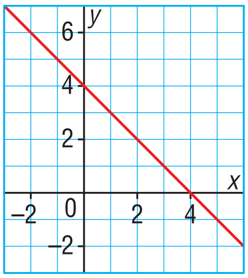
iii)



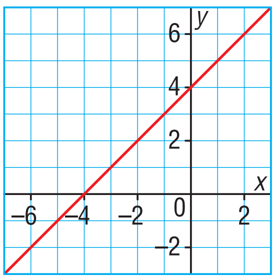
6. Match each equation with a graph below. Justify your answers.

a) $x + y = 4$ **b)** $x - y = 4$ **c)** $x - y = -4$

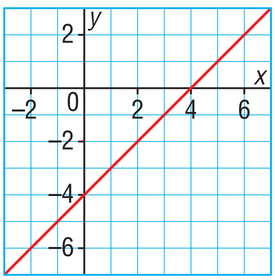
i)



ii)

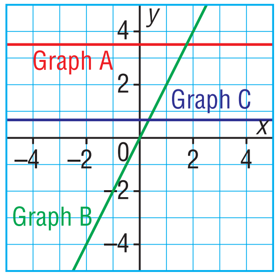


iii)

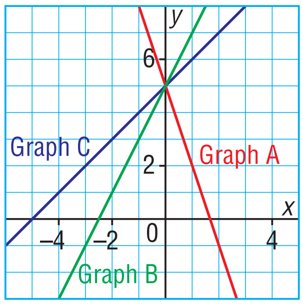


7. Match each equation with its graph below. Explain your strategy.

a) $y = 2x$ **b)** $2y = 7$ **c)** $3y = 2$

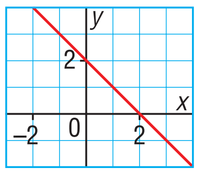


8. Which graph on this grid has equation $y = 2x + 5$? Justify your answer.

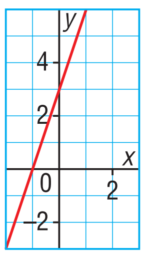


9. Which equation describes each graph? Justify your answers.

a) **i)** $y = 2x + 1$ **ii)** $y = 2x + 3$
iii) $y = x - 2$ **iv)** $y = -x + 2$



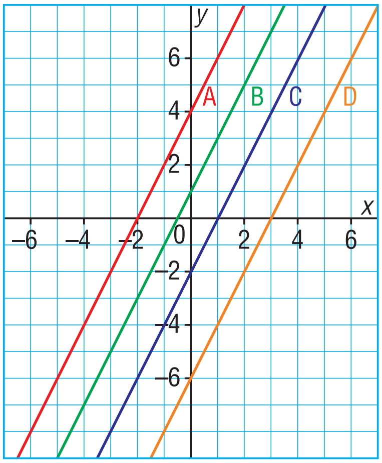
b) **i)** $x + 3y = 1$ **ii)** $3x - y = -3$
iii) $3x + y = 1$ **iv)** $3x - y = 3$



10. a) Write the equations of 3 different lines.
b) Graph the lines on the same grid. Write the equations below the grid.
c) Trade grids with a classmate. Match your classmates' graphs and equations.

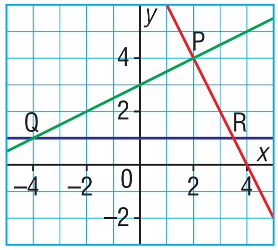
11. **Assessment Focus**

a) How are these 4 graphs alike?



b) How are the graphs different?
c) Match each graph to its equation.
i) $y = 2x - 2$
ii) $y = 2x + 4$
iii) $2x - y = 6$
iv) $2x - y = -1$
d) Did you use the same strategy each time?
If your answer is yes, what strategy did you use and why?
If your answer is no, explain why you used different strategies.
Show your work.

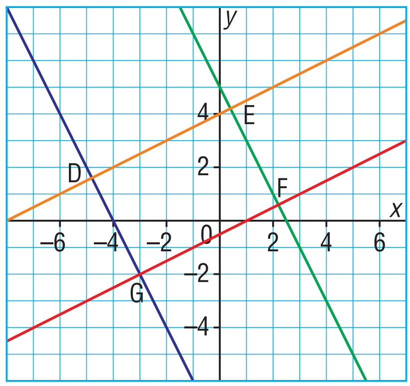
12. The lines on the grid below intersect to form ΔPQR. The equations of the lines are: $y = 1$, $2x + y = 8$, and $2y - x = 6$



What is the equation of the line on which each side of the triangle lies?
a) PQ   b) QR   c) RP

Take It Further

13. The lines on the grid below intersect to form rectangle DEFG.
The equations of the lines are: $y = \frac{1}{2}x - \frac{1}{2}$; $y = -2x + 5$; $y = -2x - 8$; and $x - 2y = -8$



What is the equation of the line on which each side of the rectangle lies?
a) DE   b) DG   c) EF   d) FG

Reflect

What strategies have you learned to match an equation with its graph?
When might you use each strategy? Include examples in your explanation.

4.5 Using Graphs to Estimate Values

How do you think city planners can predict the volume of water that will be needed by its residents in the future?

FOCUS

- Use interpolation and extrapolation to estimate values on a graph.

Investigate

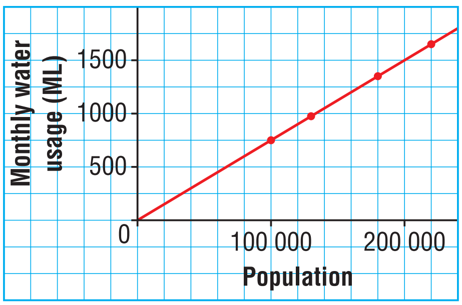
2

A city has grown over the past few years. This table and graph show how the volume of water used each month is related to the population.

Population	Monthly Water Usage (ML)
100 000	750
130 000	975
180 000	1350
220 000	1650

1 ML is 1 000 000 L.

Water Usage in One City



Use these data to:

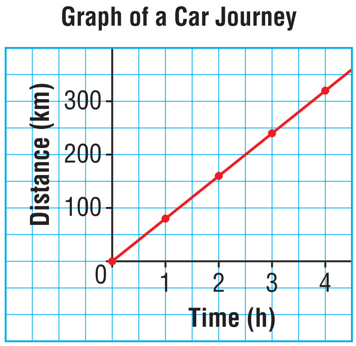
- Estimate the monthly water usage for a population of 150 000 people.
- Estimate the population when the monthly water usage is 1400 ML.
- Predict the water usage for 250 000 people.

Reflect & Share

Share your answers and strategies for solving the problems with another pair of students.
Did you use the table to estimate? Did you use the graph?
Are your estimates the same? Should they be? Explain.
Why do we call these numbers "estimates"?

Connect

This graph shows how the distance travelled by a car on the highway changes over a 4-h period.



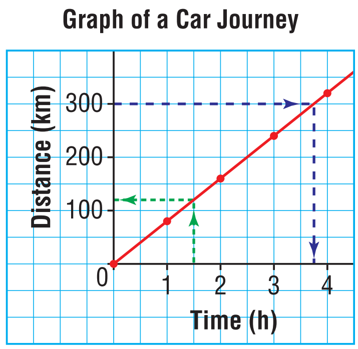
To draw the graph, we plotted the distance travelled every hour, then drew a line through the points.

We can use **interpolation** to estimate values that lie *between* 2 data points on the graph.

To estimate the distance travelled in 1.5 h:

- Begin at 1.5 on the *Time* axis.
- Draw a vertical line to the graph.
- Then draw a horizontal line from the graph to the *Distance* axis.
  This line intersects the axis at about 120 km.

So, the distance travelled in 1.5 h is about 120 km.



To estimate the time it takes to travel 300 km:

- Begin at 300 on the *Distance* axis.
- Draw a horizontal line to the graph.
- Then draw a vertical line from the graph to the *Time* axis.
  This line intersects the axis at about 3.75 h, which is 3 h 45 min.

So, it takes about 3 h 45 min to travel 300 km.

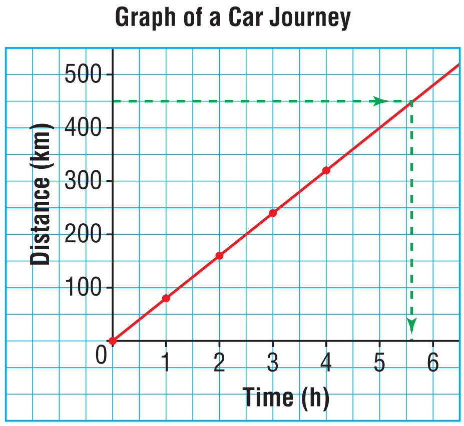
Suppose the car maintains the same average speed. We can extend the graph to predict how far the car will travel in a given time or to predict the time it takes to travel a given distance.

This is called **extrapolation.** When we use a graph to predict in this way, we assume that the relation is linear and will continue to be linear.
We use a ruler to extend the graph.

To estimate the time it takes to travel 450 km:

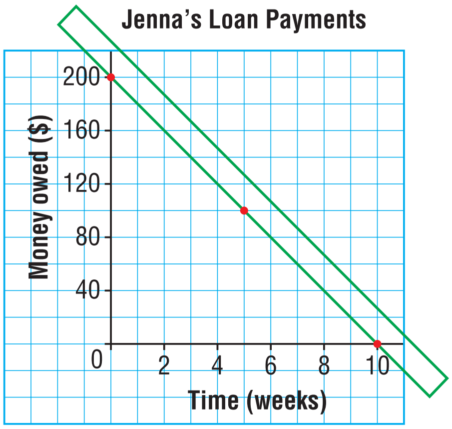
- Extend the grid so the *Distance* axis shows at least 450 km.
  Use a ruler to extend the graph.
- Repeat the process to estimate the time to travel 450 km.

It takes a little more than 5.5 h, or about 5 h 40 min to travel 450 km.



Example 1 Using Interpolation to Solve Problems

Jenna borrows money from her parents for a school trip. She repays the loan by making regular weekly payments. The graph shows how the money is repaid over time. The data are discrete because payments are made every week.



a) How much money did Jenna originally borrow?

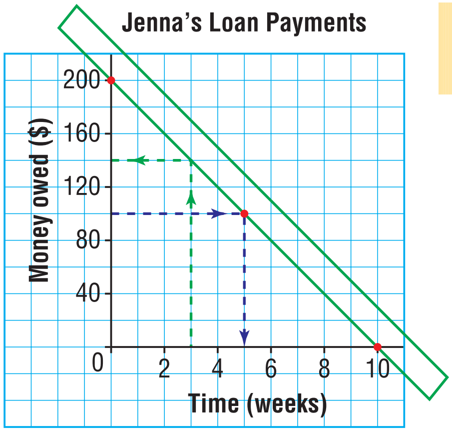
b) How much money does she still owe after 3 weeks?

c) How many weeks will it take Jenna to repay one-half of the money she borrowed?

A Solution

a) The money borrowed is the amount when the repayment time is 0.
This is the point where the graph intersects the *Money owed* axis.
Jenna originally borrowed $200.

b) Begin at 3 on the *Time* axis.
Draw a vertical line to the graph, then a horizontal line to the *Money owed* axis.
The amount owed is about halfway between 120 and 160.
So, Jenna owes about $140 after 3 weeks.

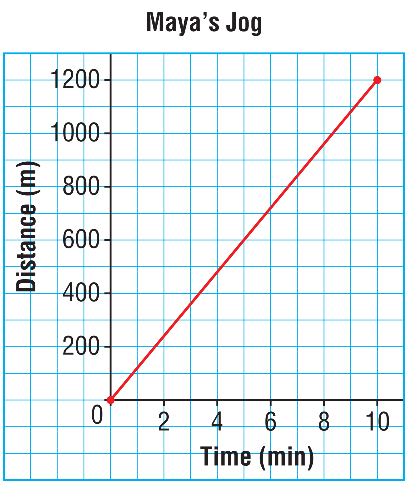


Use a straightedge to help.

c) Jenna borrowed $200.
After she repays one-half of this amount, she still owes $100.
Begin at 100 on the *Money owed* axis.
Draw a horizontal line to the graph, then a vertical line to the *Time* axis.
It will take Jenna about 5 weeks to repay one-half of the money.

Example 2 Using Extrapolation to Solve Problems

Maya jogs on a running track. This graph shows how far she jogs in 10 min.
Assume Maya continues to jog at the same average speed.



Use the graph.

a) Predict how long it will take Maya to jog 2000 m.

b) Predict how far Maya will jog in 14 min.

c) What assumption did you make?

A Solution

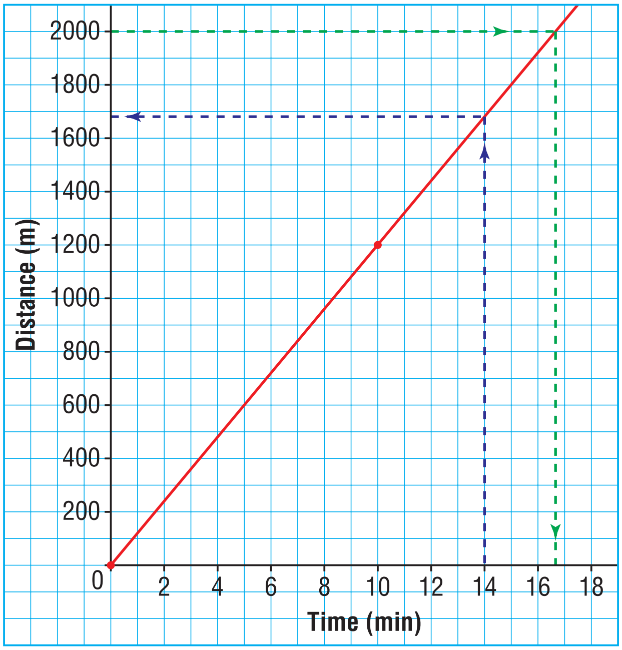
Extend the graph to include 2000 m vertically and 14 min horizontally.

a) Begin at 2000 on the *Distance* axis.
Move across to the graph then down to the *Time* axis.
It will take Maya between 16 and 17 min to jog 2000 m.

b) Begin at 14 on the *Time* axis.
Move up to the graph then across to the *Distance* axis.
The distance is about 1700 m.
In 14 min, Maya will jog about 1700 m.

c) I assume that Maya will continue to jog at the same average speed as before.

Maya's Jog

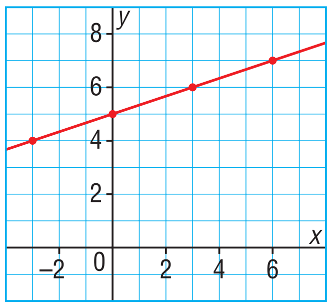


Example 3 Interpolating and Extrapolating to Determine Values of Variables from a Graph

Use this graph of a linear relation.

a) Determine the value of x when $y = 3$.

b) Determine the value of y when $x = 5$.

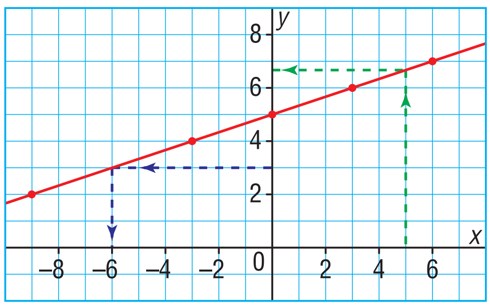


A Solution

Extend the graph to the left to be able to extrapolate for $y = 3$.
Label the extended x-axis.

a) Begin at 3 on the y-axis.
Move across to the graph, then down to the x-axis.
When $y = 3$, $x = -6$

b) Begin at 5 on the x-axis.
Move up to the graph, then across to the y-axis.
The value of y is between 6 and 7, but closer to 7.
When $x = 5$, $y \doteq 6\frac{2}{3}$



Discuss the ideas

1. a) What is interpolation? When do we use it?
   b) What is extrapolation? When do we use it?

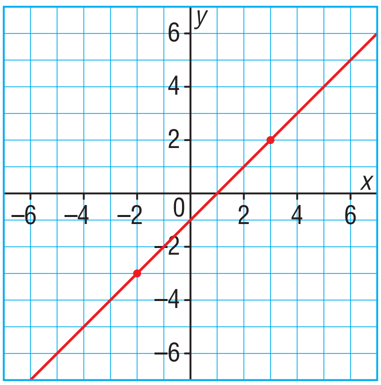
2. When we extrapolate, why is it important to know that the data represent a linear relation?

3. What problems might there be if you extrapolate far beyond the last data point?

Practice

Check

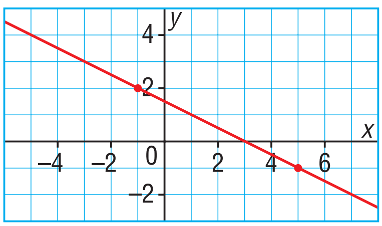
4. This graph represents a linear relation.



a) Determine each value of x for:
i) $y = 5$ ii) $y = -1$ iii) $y = -2$

b) Determine each value of y for:
i) $x = -4$ ii) $x = 2$ iii) $x = 5$

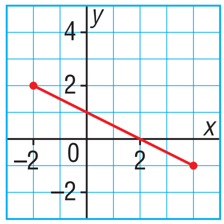
5. This graph represents a linear relation.



a) Determine each value of x for:
i) $y = 3$ ii) $y = 1$ iii) $y = -2$

b) Determine each value of y for:
i) $x = -3$ ii) $x = 3$ iii) $x = 6$

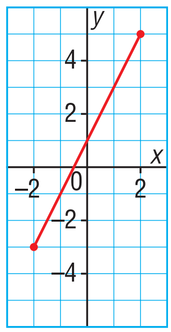
6. This graph represents a linear relation.



a) Determine each value of x for:
i) $y = 6$ ii) $y = -4$ iii) $y = -8$

b) Determine each value of y for:
i) $x = -6$ ii) $x = 6$ iii) $x = 9$

7. This graph represents a linear relation.



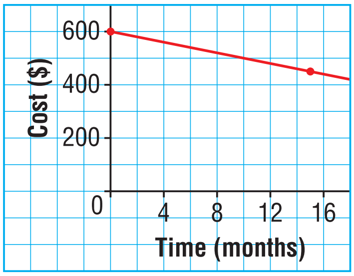
a) Determine each value of x for:
i) $y = 6$ ii) $y = -4$ iii) $y = -7$

b) Determine each value of y for:
i) $x = -5$ ii) $x = 3$ iii) $x = 5$

Apply

8. This graph shows how the price of a new game console changes with time.

Cost of a Game Console



Use the graph.

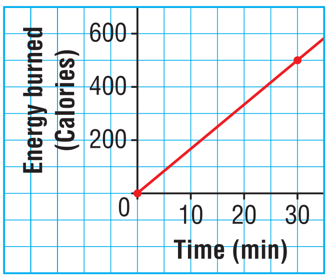
a) Estimate the cost of the game console 5 months after it is released.

b) How many months is it until the console costs $500?

c) Estimate the price of the console one year after it was released.

9. This graph shows the energy in Calories that Kendall burns when he works out on an elliptical machine.

Energy Burned on an Elliptical Machine



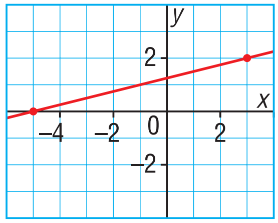
Use the graph.

a) Estimate how many Calories Kendall burns in 20 min.

b) Estimate for how long Kendall must exercise to burn 400 Calories.

c) Estimate how many Calories Kendall burns in 6 min.

10. This graph represents a linear relation.



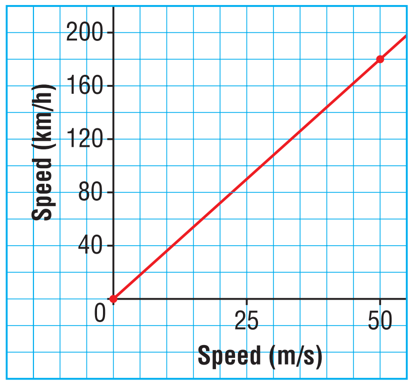
Estimate the value of y when:

a) $x = -3$ **b)** $x = 0$ **c)** $x = 1$

Explain how you estimated.

11. Assessment Focus This graph shows how a speed in metres per second relates to a speed in kilometres per hour.

Graph for Converting Speeds



a) Estimate the speed, in metres per second, of:

i) a car that is travelling at 70 km/h

ii) a train that is travelling at 110 km/h

b) Estimate the speed, in kilometres per hour, of:

i) a racing car that is travelling at 60 m/s

ii) a bicycle that is travelling at 8 m/s

c) For which of parts a and b did you use:

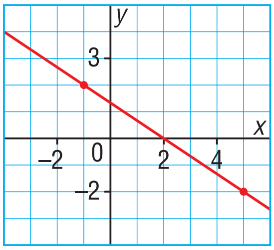
i) interpolation?

ii) extrapolation?

Explain how you know.

d) Explain why your answers are estimates and not exact.

12. This graph represents a linear relation.



Estimate the value of x when:

i) $y = 3$

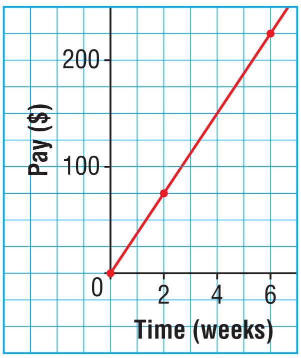
ii) $y = 1$

iii) $y = -1$

Explain how you estimated.

13. Reece works for 5 h each week at a clothing store. This graph shows how her pay relates to the number of weeks she works.

Reece's Pay

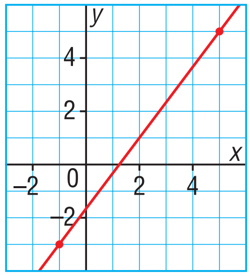


a) Estimate Reece's earnings after 8 weeks.

b) Estimate how long it will take Reece to earn $400. What assumption did you make?

c) What conditions could change that would make this graph no longer valid?

14. This graph represents a linear relation.



a) Estimate the value of y when:

i) $x = -3$ ii) $x = -5$ iii) $x = 10$

b) Estimate the value of x when:

i) $y = -5$ ii) $y = 8$ iii) $y = 10$

Explain how you estimated.

Take It Further

15. A local convenience store sells 3 different sizes of drinks. The price of each drink is listed below. The store owner plans to introduce 2 new sizes of drinks. She wants the prices and sizes to be related to the drinks she sells already.

Size (mL)	Price (¢)
500	79
750	89
1000	99

a) Graph the data.

b) What should the store owner charge for a 1400-mL drink?

c) What should be the size of a drink that costs 65¢?

Justify your answers.

Reflect

What is the difference between interpolation and extrapolation? When might you use each process? Use examples in your explanation.

Interpolating and Extrapolating

FOCUS

- Use a graphing calculator to create a graph, then interpolate and extrapolate values.

We can use a graphing calculator to graph the data in a table of values. We can then interpolate and extrapolate from the graph to estimate or predict values that are not in the table.

The table at the right shows the costs of gas for 5 customers at a gas station.
What is the cost of 20 L of gas?
How much gas can be bought for $20.00?
What is the cost of 30 L of gas?

Volume of Gas (L)	Cost ($)
6	5.10
10	8.50
16	13.60
18	15.30
24	20.40

To graph the relation:

- ➢ Enter the data in a graphing calculator.
- ➢ Set up the calculator to plot the points.
- ➢ Display the graph.

To interpolate or extrapolate:

- ➢ To determine the cost of 20 L of gas, use the table feature or trace along the graph to interpolate.
  The cost of 20 L of gas is $17.00.
- ➢ To determine how much gas can be bought for $20.00, find where the horizontal line $y = 20$ meets the graph. Input the equation $y = 20$, then use the trace or intersection feature to determine the coordinates of the point where the lines intersect.
  About 23.5 L of gas can be bought for $20.00.
- ➢ To determine the cost of 30 L of gas, extend the table or trace along the graph to extrapolate. You may need to adjust the window before you trace along the graph.
  The cost of 30 L of gas is $25.50.

Check

Follow the steps above to graph the data.

1. Use the graph to estimate each value.
  a) the cost of:
    i) 10 L of gas **ii)** 50 L of gas
  b) the volume of gas that can be purchased for:
    i) $65.00 **ii)** $12.00
  Did you interpolate or extrapolate to determine these values? Explain.

Study Guide

Generalize a Pattern

Term Number, n	Term Value, v	Pattern
1	3	$2(1) + 1$
2	5	$2(2) + 1$
3	7	$2(3) + 1$
⋮	⋮	⋮
n		$2(n) + 1$

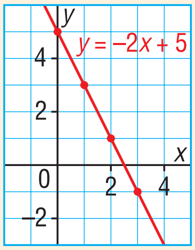
Each term value is 2 more than the preceding term value.
Start with the expression $2n$ and adjust it as necessary to produce the numbers in the table.
The expression is: $2n + 1$
The equation is: $v = 2n + 1$

Linear Relations

- The graph of a linear relation is a straight line.
  To graph a linear relation, first create a table of values.
  For example, to graph the linear relation: $y = -2x + 5$

x	y
0	5
1	3
2	1

Choose 3 values of x, then use the equation to calculate corresponding values of y.

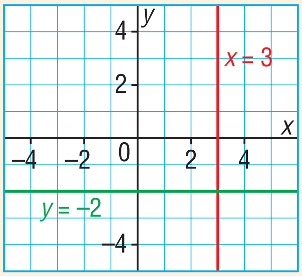


Each point on the graph is 1 unit right and 2 units down from the preceding point.

Another form of the equation of the graph above is $2x + y = 5$.

Horizontal and Vertical Lines

- The graph of the equation $x = a$, where a is a constant, is a vertical line.
  The graph of the equation $y = a$, where a is a constant, is a horizontal line.



Interpolation and Extrapolation

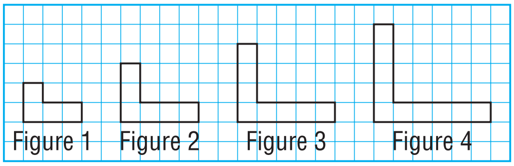
- Interpolation is determining data points *between* given points on the graph of a linear relation.

  Extrapolation is determining data points *beyond* given points on the graph of a linear relation.
  When we extrapolate, we assume that the linear relation continues.

Review

4.1

1. This pattern continues.



a) Determine the perimeter of each figure.

b) Draw the next 3 figures on grid paper.

c) Make a table to show the number of each figure and its perimeter.

d) Write an expression for the perimeter in terms of the figure number, *n*.

e) Write an equation that relates the perimeter *P* to *n*.

f) Determine the perimeter of figure 30.

g) Determine the figure number that has perimeter 90 units.

2. The pattern in this table continues.

Term Number, *n*	Term Value, *v*
1	−5
2	−2
3	1
4	4

a) Describe the patterns in the table.

b) Use *n* to write an expression for the term value.

c) Write an equation that relates *v* and *n*.

d) Verify the equation by substituting a pair of values from the table.

e) Determine the value of the 21st term.

f) Which term number has a value of 106? How do you know?

3. The first number in a pattern has the value 75. As the term number increases by 1, its value decreases by 4.

a) Create a table for this pattern.

b) Write an expression for the value of the term in terms of the term number *n*.

4.2

4. Norman has $140 in his savings account. Each month he deposits $20 into this account. Let *t* represent the time in months and *A* the account balance in dollars.

a) Create a table to show several values of *t* and *A*.

b) Graph the data. Will you join the points? Explain.

c) Is this relation linear? Justify your answer.

d) Describe the pattern in the table. How are these patterns shown in the graph?

e) Write an equation that relates *A* and *t*.

5. Copy and complete each table of values. Describe the patterns in the table.

a) $y = 4x$ **b)** $y = 10 - 2x$ **c)** $y = 3x + 4$

x	*y*
1	
2	
3	

x	*y*
0	
1	
2	

x	*y*
−3	
−2	
−1	

6. Graph the data from each table in question 5. For each graph, explain how the patterns in the graph match the patterns in the table.

4.3

7. A piece of string is 25-cm long. The string is cut into 2 pieces.

a) Make a table that shows 6 possible lengths for the two pieces of string.

b) Graph the data.

i) Is the relation linear? How do you know?

ii) Should you join the dots? Explain.

c) Choose 2 variables to represent the lengths of the longer and shorter pieces.

i) Write an equation that relates the variables.

ii) How could you check your equation?

8. Graph each equation. Do you need to make a table of values each time? Explain.

a) $x = -2$ b) $y = 3$

c) $x = 5$ d) $y = -1$

9. For each equation below:

- Make a table for the given values of x.
- Graph the equation.

a) $3x + y = 9$; for $x = -3, 0, 3$

b) $2x - y = 4$; for $x = -2, 0, 2$

c) $2x + y = -6$; for $x = -4, 0, 4$

d) $x - 2y = -6$; for $x = -2, 0, 2$

10. Does each equation represent a vertical line, a horizontal line, or an oblique line? How can you tell without graphing?

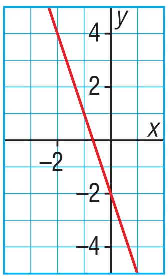
a) $x = 6$ b) $x - y = 3$

c) $y + 8 = 0$ d) $2x + 9 = 0$

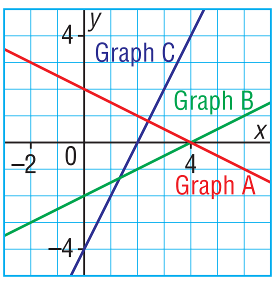
4.4 11. Which equation describes the graph below? Justify your answer.

a) $y = -2x + 3$ b) $y = 2x - 3$

c) $y = 3x - 2$ d) $y = -3x - 2$



12. Which graph represents the equation $x - 2y = 4$? How do you know?



13. Match each equation with its graph below. Explain your strategy.

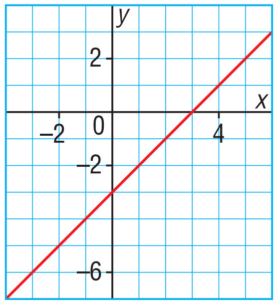
a) $x + 2y = 6$

b) $y = x - 3$

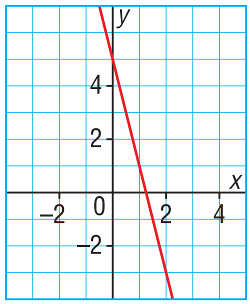
c) $y = 2x - 3$

d) $y = -4x + 5$

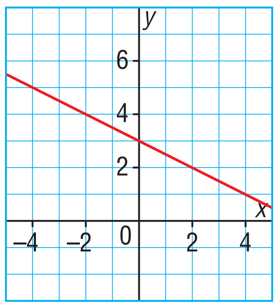
i)



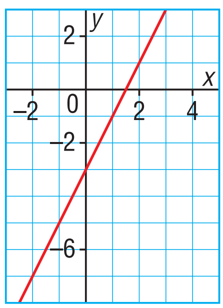
ii)



iii)

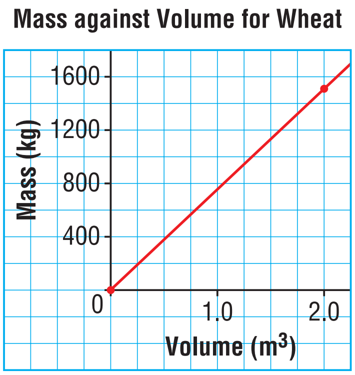


iv)



4.5

14. This graph shows how the mass of wheat changes with its volume.

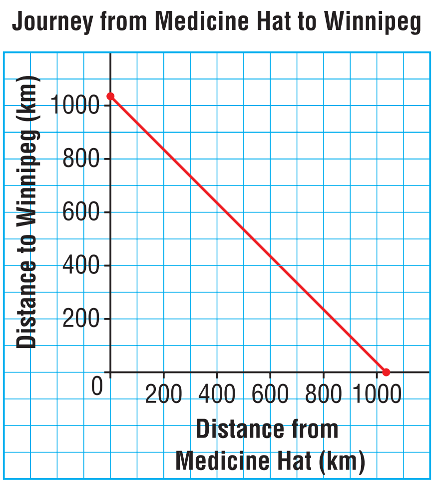


Use the graph.

a) Estimate the volume of 2000 kg of wheat.

b) Estimate the mass of 2.5 m^3 of wheat.

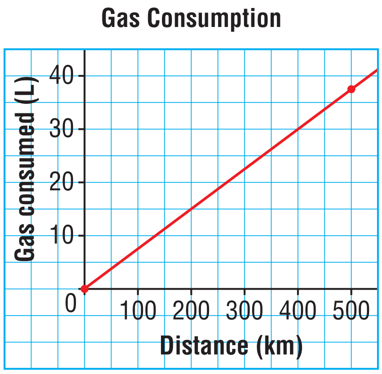
15. Harold and Jenny are driving from Medicine Hat to Winnipeg. The graph shows the distance travelled and the distance yet to go.



a) About how far is it from Medicine Hat to Winnipeg? How can you tell from the graph?

b) When Jenny and Harold have travelled 450 km, about how far do they still have to go?

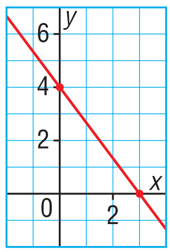
16. The Dubois family lives in Regina. The family is planning a family holiday to the West Coast. This graph shows the gas consumption of the family's car.



a) The distance from Regina to Vancouver is 1720 km. Estimate the volume of gasoline needed to travel from Regina to Vancouver. Explain how you did this.

b) To travel from Regina to Prince Albert, the car used about 30 L of gasoline. About how far is it between these two towns?

17. This graph represents a linear relation.



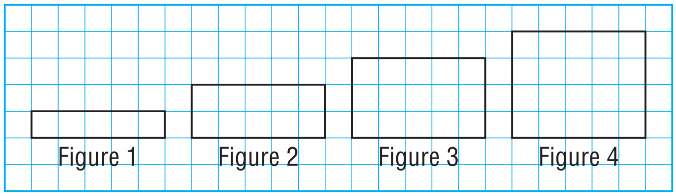
a) Estimate the value of y when:
   i) $x = -4$
   ii) $x = 2$
   iii) $x = 5$

b) Estimate the value of x when:
   i) $y = 7$
   ii) $y = 2$
   iii) $y = -3$

Explain how you estimated.

Practice Test

1. Here is a pattern made from square tiles.

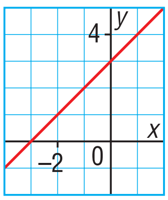


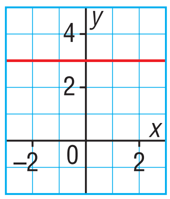
   a) Make a table that shows how the number of square tiles, s, in a figure relates to the figure number, f.
   b) Write an expression for the number of square tiles in terms of f.
   c) Write an equation that relates s and f. Verify the equation by substituting the values from the table.
   d) How are the expression and equation alike? How are they different?
   e) Which figure has 225 tiles? Explain how you know.

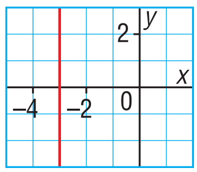
2. a) Make a table of values for this equation: $y = -2x + 7$
   b) Graph the relation.
   c) Explain how the patterns in the graph match those in the table.

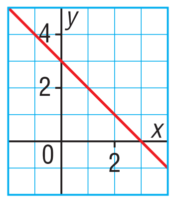
3. Does each equation describe a vertical, a horizontal, or an oblique line? How do you know?
   a) $x = 6$ b) $2y - 7 = 3$ c) $2x + 9 = 0$

4. Match each equation with its graph below. Explain your strategy.
   a) $y = x + 3$ b) $y = 3$ c) $x + y = 3$ d) $x = -3$

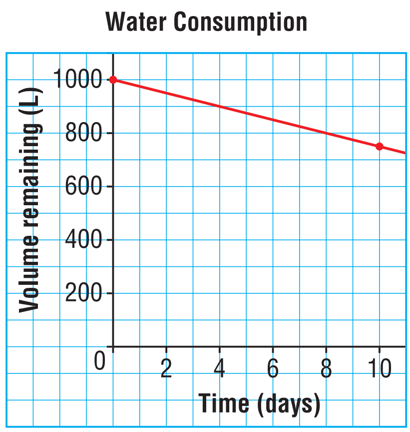
i)


ii)


iii)


iv)


5. A family uses a cistern for drinking water at its cabin. The graph shows how the volume of drinking water in the cistern changes during a 10-day period. Suppose the pattern in the water usage continues.
   a) How many days did it take to use 200 L of water?
   b) Estimate the volume of water in the cistern after 22 days.
   c) Estimate how much water is used in the first 14 days.
   d) What assumptions did you make?



Unit Problem — Predicting Music Trends

The format in which music is produced and sold has changed over the past 30 years.

Part 1

The table shows the sales of cassette tapes in North America.

Year	Cassette Sales (billions)
1993	$2.9
1995	$2.3
1998	$1.4

a) Graph the data. Do the data represent a linear relation? How do you know?

b) Describe how the sales of cassettes changed over time.

c) Let t represent the number of years after 1993 and S the sales in billions of dollars. Write an equation that relates S and t.

d) Use the equation to determine the sales in 1997. Does the answer agree with the value in the graph? Explain.

e) Use the graph to predict the year in which the sales of cassettes were $0.

f) Cassettes were sold until 2004. Explain why this is different from the year predicted in the graph.

Part 2

As the sale of cassettes was decreasing, the sales of CDs were increasing. Assume the growth in CDs sales, from 1996 to 2000, was linear.

Year	CD Sales (millions)
1996	$9 935
2000	$13 215

a) Graph the data. Use the graph to estimate the CD sales for 1997, 1998, and 1999. Is this interpolation or extrapolation? Explain.

b) Estimate the total CD sales for this 5-year period.

c) Estimate the CD sales in 2001. Is this interpolation or extrapolation? Explain.

d) Use the graph to estimate the CD sales for 2005.

e) Which answer in parts c and d is more likely to be the closer estimate? Justify your answer.

Your work should show:

- accurate and labelled graphs
- how you wrote and used the equation
- clear explanations of your thinking

Reflect on Your Learning

What is a linear relation? How may a linear relation be described? What can you determine when you know a relation is linear? Include examples.

Project

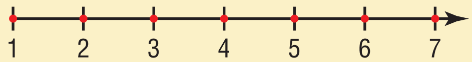
Number Systems

Materials
- copies of a Frayer model

Work with a partner.

Part 1

This is a Frayer model for **natural numbers**.

Definition	**Characteristics**
the set of numbers that includes all the whole numbers greater than or equal to 1	• "counting numbers" • represented with dots on a number line 
Natural Numbers	
Examples 1 2 3 4 16 5 000 000 6 5	**Non-examples** -32 $3\frac{4}{5}$ 4.125 $\sqrt{2}$ $0.\overline{3}$

➤ Complete a Frayer model for each of these number systems:
- whole numbers
- integers
- rational numbers

Each of these number systems is part of a larger system of numbers called the **real numbers**.

Part 2

➤ Is it possible for a number to belong to more than one number system? Explain. Use examples to support your answer.

➤ Is it possible for an entire number system to belong to another number system? Explain.

➤ Draw a diagram to show how the number systems in Part 1 overlap.

Part 3

Choose a number system from Part 1; this is the name of your club.
Choose a number from your number system.

You are in charge of memberships for your club. It is your job to either accept or reject a number that wishes to join your club. Write a letter of acceptance or rejection for your partner's number. If the number belongs in your number system, it must be allowed membership.

Your letter should be written as a business letter.
It must address the following points:

- examples of other numbers in your club and what their characteristics are
- how your partner's number either fits or does not fit the characteristics of your club
- if you are accepting the number, why your club wants that number
- if you are rejecting the number, what other clubs the number could contact and why

Take It Further

- Numbers that are not rational numbers are called *irrational numbers*. Create a Frayer model for *irrational numbers*.
- Amend your diagram from Part 2 to include these numbers.